

Inflation from Superstring/M-Theory Compactification with Higher Order Corrections II – Case of Quartic Weyl Terms –

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Abstract

We present a detailed study of inflationary solutions in M-theory with higher order quantum corrections. We first exhaust all exact and asymptotic solutions of exponential and power-law expansions in this theory with quartic curvature corrections, and then perform a linear perturbation analysis around fixed points for the exact solutions in order to see which solutions are more generic and give interesting cosmological models. We find an interesting solution in which the external space expands exponentially and the internal space is static both in the original and Einstein frames. Furthermore, we perform a numerical calculation around this solution and find numerical solutions which give enough e-foldings. We also briefly summarize similar solutions in type II superstrings.

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1 Introduction

The recent cosmological observations have confirmed the existence of the early inflationary epoch and the accelerated expansion of the present universe [1]. Though it is not difficult to construct cosmological models with these features if one introduces scalar fields with suitable potentials, it is desirable to derive such a model from fundamental theories of particle physics without making special assumptions. The most promising candidates for such theories are the ten-dimensional superstrings or eleven-dimensional M-theory, which are hoped to give models of accelerated expansion of the universe upon compactification to four dimensions.

There is a no-go theorem which forbids such solutions if six- or seven-dimensional internal space is a time-independent nonsingular compact manifold without boundary [2]. This theorem also assumes that the gravity action does not contain higher curvature corrections. So we can evade this theorem by violating some of the assumptions. In fact, it has been shown that a model with certain period of accelerated expansion can be obtained from the higher-dimensional vacuum Einstein equation if one assume a time-dependent hyperbolic internal space [3]. It has been shown [4] that this class of models is obtained from what are known as S-branes [5, 6, 7] in the limit of vanishing flux of three-form fields (see also [8]). For other attempts for inflation in the context of string theories, see, for instance, Refs. [9, 10, 11].

The scale when the acceleration occurs in this type of models is basically governed by the Planck scale in the higher ten or eleven dimensions. With phenomena at such high energy, it is expected that we cannot ignore quantum corrections such as higher derivative terms in the theories at least in the early universe. It is known that there are terms of higher orders in the curvature to the lowest effective supergravity action coming from superstrings or M-theory [12, 13, 14, 15, 16]. The no-go theorem does not apply to theories with higher derivatives because they violate the assumptions. With such corrections, they will significantly affect the inflation at the early stage of the evolution of our universe.

The cosmological models in higher dimensions were studied intensively in the 80's by many authors [17, 18, 19, 20, 21]. It was shown that inflation is indeed possible with higher-order curvature corrections [19, 20]. However, most of the work considered theories with higher orders of scalar curvature, which are not typical correction terms known to arise in the superstring theories or M-theory. It is thus important to examine if the above result of small e-folding is modified with higher-order corrections expected in these fundamental theories.

In particular, the leading quadratic correction for heterotic string theories is proportional to the Gauss-Bonnet (GB) combination [12, 13, 14]. This model has been studied in some detail in Ref. [19] and it was shown that there are two exponentially expanding solutions, which may be called generalized de Sitter solutions since the size of the internal space also depends on time. Despite the exponential behavior in the original frame, those solutions give non-inflationary power-law expansion in the Einstein frame as indicated in our previous papers [22, 23]. On the other hand, there is no quadratic and cubic curvature corrections in type II superstring theory or M-theory, and thus the first higher order corrections start with quartic curvature terms [15, 16].

In our previous papers [22, 23], we have reported our results on this problem for M-theory with forth-order terms in terms of the Riemann curvature tensors. However, it turns out that the coefficients we took had opposite signs to the M-theory case. Since there is ambiguity in the coefficients coming from the field redefinition, this does not immediately invalidate the results, but it is necessary to study how the results change. Here we give the details of our results in M-theory as well as type II superstrings with the correct coefficients. We consider these corrections given in terms of the Weyl tensors which are favorable because only corrections in this scheme do not affect the highly symmetric tree-level solutions such as $\text{AdS}_7 \times S^4$ [16]. We also include an additional quartic term in scalar curvature with an arbitrary coefficient δ in order to take into account the ambiguity mentioned above. With this action, we exhaust exact solutions as well as past and future asymptotic solutions and then discuss inflationary solutions among them. The past and future asymptotic solutions are useful in describing the inflation at the early universe and the present accelerating cosmology, respectively. Furthermore, we perform a linear perturbation analysis in order to see which solutions are more generic and to make interesting cosmological models. We find an interesting solution in which the external space expands exponentially and the internal space is static both in the original and Einstein frames. This may be regarded as “moduli stabilization” by higher order corrections, but this is not the usual moduli stabilization in the sense that the moduli are fixed and stable. Rather we are interested in such solutions in

which the sizes of the internal spaces do not grow too much while they exhibit inflation. What we find is that this is possible with higher order corrections. We also perform a numerical calculation around this solution and find that some spacetimes give enough e-foldings. Finally we briefly summarize similar solutions in type II superstrings. Necessity of the higher order corrections for inflation in M-theory is also discussed in Ref. [24].

In the next section, we present our actions and field equations to be solved. We write down these for $D = (1 + p + q)$ dimensions with p external and q internal space dimensions. Though we are mainly interested in $p = 3$ in this paper, there may be interesting applications if we keep the dimension p arbitrary. We give the equations for maximally symmetric spaces with non-vanishing curvatures. The explicit forms of the actions are given in Appendix A, and the field equations in Appendices B and C. Although similar equations are given in our previous papers [22, 23], the present theory is different in that the quartic terms are written in the Weyl tensor instead of Riemann, and also that we have an additional term R^4 . So we present these equations for our new system.

We give exact solutions as well as past and future asymptotic solutions with exponential and power-law expansions in § 3 for $\delta = 0$ and in § 4 for $\delta \neq 0$. Also we present the solutions for maximally symmetric spaces with non-vanishing curvatures.

In § 5, we perform a linear perturbation analysis around fixed points for the exact generalized de Sitter solutions that are given in § 3 and § 4 in order to see which solutions are more generic and to make interesting cosmological scenario.

In § 6, we also summarize exact solutions for type IIB superstrings for constant dilaton.

Using the obtained solutions, we discuss an inflationary scenario in § 7. Many of our exact solutions do not seem to give successful inflation in the sense that they do not give big enough e-foldings. However the simple analysis of exact solutions does not tell us what happens after the inflationary solutions decay. Actually it turns out by numerical analysis that there are interesting solutions, which first approach to the exact solution for which the external space expands exponentially and the internal space is static both in the original and Einstein frames, and then eventually go to a stable solution. For such solutions, we show that it is possible to obtain enough e-foldings for successful inflation. This is a very interesting possibility of achieving inflationary solutions.

The contents of §§ 3 – 6 summarize our analysis of solutions and are rather technical. The reader may find it useful to skip this part for the first reading, and then come back to check when they study physically interesting solutions described in § 7.

2 Field equations

We consider the low-energy effective action for M-theory ($D = 11$) with higher order corrections keeping dimension D arbitrary:

$$S = \sum_{n=1}^4 S_n + S_W + S_{R^4}, \quad (2.1)$$

with

$$S_1 = S_{\text{EH}} \equiv \frac{\alpha_1}{2\kappa_D^2} \int d^D x \sqrt{-g} R, \quad (2.2)$$

$$S_2 = S_{\text{GB}} \equiv \frac{\alpha_2}{2\kappa_D^2} \int d^D x \sqrt{-g} [R_{\mu\nu\rho\sigma}^2 - 4R_{\mu\nu}^2 + R^2], \quad (2.3)$$

$$S_3 = \frac{\alpha_3}{2\kappa_D^2} \int d^D x \sqrt{-g} \tilde{E}_6, \quad (2.4)$$

$$S_4 = \frac{\alpha_4}{2\kappa_D^2} \int d^D x \sqrt{-g} \tilde{E}_8, \quad (2.5)$$

$$S_W = \frac{\gamma}{2\kappa_D^2} \int d^D x \sqrt{-g} L_W, \quad (2.6)$$

$$S_{R^4} = \frac{\delta}{2\kappa_D^2} \int d^D x \sqrt{-g} R^4, \quad (2.7)$$

where

$$\tilde{E}_{2n} = -\frac{1}{2^n(D-2n)!} \epsilon^{\alpha_1 \dots \alpha_{D-2n} \mu_1 \nu_1 \dots \mu_n \nu_n} \epsilon_{\alpha_1 \dots \alpha_{D-2n} \rho_1 \sigma_1 \dots \rho_n \sigma_n} R^{\rho_1 \sigma_1}_{\mu_1 \nu_1} \dots R^{\rho_n \sigma_n}_{\mu_n \nu_n}, \quad (2.8)$$

$$L_W = C^{\lambda\mu\nu\kappa} C_{\alpha\mu\nu\beta} C_{\lambda}^{\rho\sigma\alpha} C_{\rho\sigma\kappa}^{\beta} + \frac{1}{2} C^{\lambda\kappa\mu\nu} C_{\alpha\beta\mu\nu} C_{\lambda}^{\rho\sigma\alpha} C_{\rho\sigma\kappa}^{\beta}, \quad (2.9)$$

and R^4 is a quartic term of scalar curvature. Here we have dropped contributions from form fields, κ_D^2 is a D -dimensional gravitational constant, and $\alpha_1, \dots, \alpha_4, \gamma$ and δ are numerical coefficients. The coefficient α_1 of the Einstein-Hilbert (EH) term is 1 by definition, and though α_3 is zero for all superstrings and M-theory, we have included it since it will be useful for examining other cases. The Weyl tensors in L_W are defined by

$$C_{\lambda\mu\nu\kappa} = R_{\lambda\mu\nu\kappa} - \frac{1}{D-2} (g_{\lambda\nu} R_{\mu\kappa} - g_{\lambda\kappa} R_{\mu\nu} - g_{\mu\nu} R_{\lambda\kappa} + g_{\mu\kappa} R_{\lambda\nu}) \\ + \frac{R}{(D-1)(D-2)} (g_{\lambda\nu} g_{\mu\kappa} - g_{\lambda\kappa} g_{\mu\nu}). \quad (2.10)$$

In our previous papers [22, 23], we gave field equations for this system with arbitrary couplings, but we focused on the case of nonzero α_4 and γ . We discussed the flat external and internal spaces in Ref. [22] with various coefficients of the higher order terms, whereas all combinations of curved spaces are examined in [23]. We considered the case in which the quartic term L_W are written in terms of Riemann tensors. However, it turned out that the coefficients we took in [23] were opposite in sign to the M-theory case,¹ and we should set

$$\alpha_2 = \alpha_3 = 0, \quad \alpha_4 = \frac{\kappa_{11}^2 T_2}{3^2 \times 2^9 \times (2\pi)^4}, \quad \gamma = \frac{\kappa_{11}^2 T_2}{3 \times 2^4 \times (2\pi)^4}, \quad (2.11)$$

where $T_2 = (2\pi^2/\kappa_{11}^2)^{1/3}$ is the membrane tension. Though the field equations remain valid, the numerical results on the generalized de Sitter solutions are significantly affected by this sign change. We find that many of the solutions found in our previous paper go away if we simply reverse the signs of these coefficients. However, we should also note that contributions of the Ricci tensor $R_{\mu\nu}$ and scalar curvature R are not included in the fourth-order corrections (2.9) because these terms are not uniquely fixed. This means that there is significant ambiguity in the additional terms involving these tensors, and in particular this allows us to put the forth-order terms in terms of the Weyl tensors as given above. This form appears particularly favorable because only corrections in this scheme do not affect the highly symmetric tree-level solutions such as $\text{AdS}_7 \times S^4$ ($\text{AdS}_5 \times S^5$ for type IIB superstring theory) [16]. In view of this situation, it appears more appropriate to consider the quartic correction terms given by Weyl tensors.

Furthermore it is interesting to examine how the ambiguity may affect the results. For this purpose, we also include additional quartic Ricci scalar term (2.7). Here we discuss what value of δ is plausible. Writing down Eq. (2.9) in terms of Riemann curvature tensor, we have the following equation:

$$L_W(R_{\mu\nu\rho\sigma}) = L_W(C_{\mu\nu\rho\sigma}) + \frac{12(D-4)}{(D-1)^2(D-3)^3} R^2 R_{\alpha\beta} R^{\alpha\beta} + \frac{60}{(D-1)^2(D-3)^3} R^4 \\ + (\text{terms containing both } C_{\mu\nu\rho\sigma} \text{ and } R_{\alpha\beta}, R), \quad (2.12)$$

where we omit terms which contain both $C_{\mu\nu\rho\sigma}$ and $R_{\alpha\beta}, R$, and $L_W(R_{\mu\nu\rho\sigma})$ is defined by

$$L_W(R_{\mu\nu\rho\sigma}) = R^{\lambda\mu\nu\kappa} R_{\alpha\mu\nu\beta} R_{\lambda}^{\rho\sigma\alpha} R_{\rho\sigma\kappa}^{\beta} + \frac{1}{2} R^{\lambda\kappa\mu\nu} R_{\alpha\beta\mu\nu} R_{\lambda}^{\rho\sigma\alpha} R_{\rho\sigma\kappa}^{\beta}. \quad (2.13)$$

We thus find that the difference between L_W in terms of the Riemann and Weyl tensors depends on the Ricci and scalar curvatures with very suppressed coefficient due to large $D = 11$. It is then natural to consider the

¹See the recently replaced version of Ref. [16].

R^4 correction terms with the same order of magnitude as above. Otherwise the scalar curvature terms will be dominant and the stringy effects may not be seen. Thus, the appropriate value of δ appears to have the order

$$|\delta| \sim \frac{60}{(D-1)^2(D-3)^3} \gamma \sim 10^{-3} \gamma . \quad (2.14)$$

However, since we do not know the exact contribution from Ricci and scalar curvatures, we leave δ to be free.

This is the system that we are going to examine.

2.1 Basic equations for cosmology

The metric of our D -dimensional space is

$$ds_D^2 = -N^2(t)dt^2 + a^2(t)ds_p^2 + b^2(t)ds_q^2 , \quad (2.15)$$

with

$$N(t) = e^{u_0(t)} , \quad a(t) = e^{u_1(t)} , \quad b(t) = e^{u_2(t)} , \quad (2.16)$$

where $D = 1 + p + q$. The external p - and internal q -dimensional spaces (ds_p^2 and ds_q^2) are chosen to be maximally symmetric. The curvature constants of those spaces are defined by σ_p and σ_q . The sign of σ_p (σ_q) determines the type of maximally symmetric spaces, i.e. σ_p (or σ_q) = -1, 0 and 1 denote a hyperbolic space, a flat Euclidean space, and a sphere, respectively. The hyperbolic and flat spaces are supposed to be compactified by identifying boundaries of those finite part.

From the variation of the total action (2.1), whose explicit forms with the metric (2.15) are given in Appendix A, with respect to u_0, u_1 and u_2 , we find three basic field equations:

$$F \equiv \sum_{n=1}^4 F_n + F_W + F_{R^4} = 0 , \quad (2.17)$$

$$F^{(p)} \equiv \sum_{n=1}^4 f_n^{(p)} + X \sum_{n=1}^4 g_n^{(p)} + Y \sum_{n=1}^4 h_n^{(p)} + F_W^{(p)} + F_{R^4}^{(p)} = 0 , \quad (2.18)$$

$$F^{(q)} \equiv \sum_{n=1}^4 f_n^{(q)} + Y \sum_{n=1}^4 g_n^{(q)} + X \sum_{n=1}^4 h_n^{(q)} + F_W^{(q)} + F_{R^4}^{(q)} = 0 , \quad (2.19)$$

where $X = \ddot{u}_1 - \dot{u}_0 \dot{u}_1 + \dot{u}_1^2$, $Y = \ddot{u}_2 - \dot{u}_0 \dot{u}_2 + \dot{u}_2^2$, and

$$\begin{aligned} F_n &= F_n(u_0, \dot{u}_1, \dot{u}_2, A_p, A_q) , \\ F_W &= F_W(u_0, u_1, u_2, \dot{u}_0, \dot{u}_1, \dot{u}_2, \ddot{u}_1, \ddot{u}_2, X, Y, \dot{X}, \dot{Y}) , \\ F_{R^4} &= F_{R^4}(u_0, u_1, u_2, \dot{u}_0, \dot{u}_1, \dot{u}_2, \ddot{u}_1, \ddot{u}_2, \ddot{u}_1, \ddot{u}_2, X, Y, \dot{X}, \dot{Y}) , \\ f_n^{(p)} &= f_n^{(p)}(u_0, \dot{u}_1, \dot{u}_2, A_p, A_q) , \quad g_n^{(p)} = g_n^{(p)}(u_0, \dot{u}_1, \dot{u}_2, A_p, A_q) , \\ h_n^{(p)} &= h_n^{(p)}(u_0, \dot{u}_1, \dot{u}_2, A_p, A_q) , \\ F_W^{(p)} &= F_W^{(p)}(u_0, u_1, u_2, \dot{u}_0, \dot{u}_1, \dot{u}_2, \ddot{u}_1, \ddot{u}_2, X, Y, \dot{X}, \dot{Y}, \ddot{X}, \ddot{Y}) , \\ F_{R^4}^{(p)} &= F_{R^4}^{(p)}(u_0, u_1, u_2, \dot{u}_0, \dot{u}_1, \dot{u}_2, \ddot{u}_1, \ddot{u}_2, \ddot{u}_1, \ddot{u}_2, X, Y, \dot{X}, \dot{Y}, \ddot{X}, \ddot{Y}) , \\ f_n^{(q)} &= f_n^{(q)}(u_0, \dot{u}_1, \dot{u}_2, A_p, A_q) , \quad g_n^{(q)} = g_n^{(q)}(u_0, \dot{u}_1, \dot{u}_2, A_p, A_q) , \\ h_n^{(q)} &= h_n^{(q)}(u_0, \dot{u}_1, \dot{u}_2, A_p, A_q) , \\ F_W^{(q)} &= F_W^{(q)}(u_0, u_1, u_2, \dot{u}_0, \dot{u}_1, \dot{u}_2, \ddot{u}_1, \ddot{u}_2, X, Y, \dot{X}, \dot{Y}, \ddot{X}, \ddot{Y}) , \\ F_{R^4}^{(q)} &= F_{R^4}^{(q)}(u_0, u_1, u_2, \dot{u}_0, \dot{u}_1, \dot{u}_2, \ddot{u}_1, \ddot{u}_2, \ddot{u}_1, \ddot{u}_2, X, Y, \dot{X}, \dot{Y}, \ddot{X}, \ddot{Y}) , \end{aligned} \quad (2.20)$$

are explicitly given in Appendix B. Here A_p and A_q are defined by

$$\begin{aligned} A_p &= \dot{u}_1^2 + \sigma_p \exp[2(u_0 - u_1)], \\ A_q &= \dot{u}_2^2 + \sigma_q \exp[2(u_0 - u_2)]. \end{aligned} \quad (2.21)$$

Since u_0 is a gauge freedom of time coordinate, we have three equations for two variables u_1 and u_2 . It looks like an over-determinant system. However, these three equations are not independent. In fact, we can derive the following equation after bothersome calculation:

$$\dot{F} + (p\dot{u}_1 + q\dot{u}_2 - \dot{u}_0)F = p\dot{u}_1 F^{(p)} + q\dot{u}_2 F^{(q)}. \quad (2.22)$$

The constraint equation $F = 0$ is satisfied if other dynamical equations are solved *and* it is initially satisfied. As argued in Ref. [23], it is in general enough to solve the two equations $F = 0$ and $F^{(p)} = 0$ (or $F^{(q)} = 0$) instead of trying to solve all three equations.

2.2 Ansatz for solutions

We now analyze our basic Eqs. (2.17) – (2.19) and look for inflationary solutions. Since we are interested in analytic solutions, we study the following two cases:

(1) Generalized de Sitter solutions:

Using a cosmic time, i.e. $u_0 = 0$, an exponential expansion of each scale factor is given by $u_1 = \mu t + \ln a_0$, and $u_2 = \nu t + \ln b_0$, where μ, ν, a_0 and b_0 are constants.

(2) Power-law solutions:

To find a power-law solution, although we can discuss it with the above cosmic time, we use a different time gauge, which is defined by $u_0 = t$. Using this time coordinate, a power-law solution is given by $u_1 = \mu t + \ln a_0$, and $u_2 = \nu t + \ln b_0$, where μ and ν are constants.

If we choose the following time coordinate u_0 ,

$$u_0 = \epsilon t, \quad u_1 = \mu t + \ln a_0, \quad \text{and} \quad u_2 = \nu t + \ln b_0, \quad (2.23)$$

we can discuss both solutions in the same set up, that is, $\epsilon = 0$ for case (1), while $\epsilon = 1$ for case (2). In the latter case, in terms of a cosmic time $\tau = e^t$, we see that the solution gives the power-law behavior:

$$a = e^{u_1} = a_0 \tau^\mu, \quad \text{and} \quad b = e^{u_2} = b_0 \tau^\nu. \quad (2.24)$$

Note that when the curvature constant σ_p (or σ_q) vanishes, a_0 and b_0 are arbitrary but we can set $a_0 = 1$ (or $b_0 = 1$) because such a numerical constant can be absorbed by rescaling of the spatial coordinates.

Before giving the solution, we note on the unit used in our solutions. Rescaling α_4, γ, μ and ν as

$$\tilde{\alpha}_4 = \alpha_4/|\gamma|, \quad \tilde{\gamma} = \gamma/|\gamma| (= 1), \quad \tilde{\mu} = \mu|\gamma|^{1/6}, \quad \text{and} \quad \tilde{\nu} = \nu|\gamma|^{1/6}, \quad (2.25)$$

we can always set γ to +1. We also have to rescale time coordinate as $\tilde{t} = |\gamma|^{-1/6}t$. The typical dynamical time scale is then given by $|\gamma|^{1/6} \sim O(m_D^{-1})$, where $m_D = \kappa_D^{-2/(D-2)}$ is the fundamental Planck scale. In particular, for M-theory, we find $|\gamma|^{1/6} = 6^{-1/6}(4\pi)^{-5/9}m_{11}^{-1} \sim 0.1818176m_{11}^{-1}$ from Eq. (2.11). After this scaling for the M-theory, we have

$$\tilde{\alpha}_4 = \frac{1}{3 \times 2^5}, \quad \tilde{\gamma} = 1, \quad (2.26)$$

and a free parameter $\tilde{\delta}$.

We use the above unit throughout this paper and omit tilde for simplicity. We now present solutions for $\delta = 0$ and $\delta \neq 0$ in order.

3 Solutions in M theory for $\delta = 0$

From Eq. (C.2), we expect there may exist no exact solutions expect for the case $\sigma_p = \sigma_q = 0$. However, even for the case of $\sigma_p \neq 0$ or $\sigma_q \neq 0$, we may have some asymptotic solutions either in the future direction ($t \rightarrow \infty$) or in the past direction ($t \rightarrow -\infty$), which describe the universes in these stages. We classify solutions for Eqs. (2.17) – (2.19) by the signatures of σ_p and σ_q in the following subsections.

3.1 $\sigma_p = \sigma_q = 0$

In this case, $A_p = \mu^2, A_q = \nu^2$ are constants. We will discuss the cases of $\epsilon = 0$ and $\epsilon = 1$ in order.

3.1.1 Generalized de Sitter solutions ($\epsilon = 0$)

In this case, we can take the independent equations as $F = 0$ and $F^{(p)} + F^{(q)} = 0$ for $\mu, \nu \neq 0$. From the explicit forms of field equations given in Appendix C, we have two algebraic equations for $p = 3$:

$$\begin{aligned} & \alpha_1 [6\mu^2 + 6q\mu\nu + q_1\nu^2] + \alpha_4\nu^5 [336q_4\mu^3 + 168q_5\mu^2\nu + 24q_6\mu\nu^2 + q_7\nu] \\ & + \frac{6\gamma q(\mu - \nu)^4}{(q+2)^4(q+3)^3} [-2(q-1)N_1(q, 3)\mu^4 + 12(q-1)(2q^6 + 7q^5 - 31q^4 + 39q^3 + 565q^2 - 6q - 1512)\mu^3\nu \\ & - 2(q-1)(14q^6 + 13q^5 - 242q^4 + 1079q^3 + 4014q^2 - 2970q - 7236)\mu^2\nu^2 \\ & + 4(2q^7 - 3q^6 - 11q^5 + 413q^4 + 337q^3 - 1494q^2 + 900q + 216)\mu\nu^3 + (q-7)N_1(3, q)\nu^4] \\ & + \delta(-12\mu^2 + 6q\mu\nu + q(q-7)\nu^2)[12\mu^2 + 6q\mu\nu + (q+1)_0\nu^2]^3 = 0, \end{aligned} \quad (3.1)$$

$$\begin{aligned} & (\mu - \nu) \{ \alpha_1 + 4\alpha_4 [30(q-1)_4\mu^2\nu^4 + 12(q-1)_5\mu\nu^5 + (q-1)_6\nu^6] \\ & + \frac{4\gamma(\mu - \nu)^2}{(q+2)^4(q+3)^3} [q_1N_1(q, 3)\mu^4 \\ & - 3(q-1)(2q^7 + 17q^6 - 11q^5 - 142q^4 + 1109q^3 + 1995q^2 - 3978q - 2376)\mu^3\nu \\ & + (q-1)(7q^7 + 40q^6 - 104q^5 - 114q^4 + 4887q^3 + 6372q^2 - 14580q - 7776)\mu^2\nu^2 \\ & - 2q(2q^7 + 4q^6 - 11q^5 + 135q^4 + 1104q^3 + 9q^2 - 3366q + 3024)\mu\nu^3 + 6N_1(3, q)\nu^4] \\ & + 4\delta[12\mu^2 + 6q\mu\nu + (q+1)_0\nu^2]^3 \} = 0, \end{aligned} \quad (3.2)$$

where $N_1(q, p)$ is an integer constant defined by

$$\begin{aligned} N_1(q, p) = & p^3(3p^2(q-2) + p(q^2 + 7q - 14) - 2q(2q^2 - 5q + 1) - 7) \\ & + (q-1)^3(p^2(q-3) + 3pq(q+1) - 3q^2). \end{aligned} \quad (3.3)$$

Using the values for $\tilde{\alpha}_4$ and $\tilde{\gamma}$ in Eq. (2.26) and setting $q = 7$, we can solve these equations numerically and found the following solution for $\delta = 0$:

$$(\mu, \nu) = \text{ME1}_{\pm}(\pm 0.10465, \mp 0.93666). \quad (3.4)$$

Here ME1_{\pm} (the first exact solutions in M theory) are the names given to the solutions. We will use similar names for solutions in what follows.

3.1.2 Power-law solutions ($\epsilon = 1$)

Setting $\epsilon = 1$ in Eqs. (C.4) – (C.29), there is no exact solutions. We find that the EH action is dominant as $t \rightarrow \infty$, while the actions S_4 , S_W and S_{R^4} become dominant as $t \rightarrow -\infty$. Here we present asymptotic power-law solutions for each case.

As $t \rightarrow \infty$, EH term dominates and our basic equations reduce to

$$p_1\mu^2 + q_1\nu^2 + 2pq\mu\nu = 0, \quad (3.5)$$

$$q\nu(\nu - \mu - 1) - (p - 1)\mu = 0, \quad (3.6)$$

$$p\mu(\mu - \nu - 1) - (q - 1)\nu = 0. \quad (3.7)$$

We can easily show that these three equations are equivalent to the following two equations, if it is not Minkowski space ($\mu = \nu = 0$):

$$p\mu^2 + q\nu^2 = 1, \quad p\mu + q\nu = 1, \quad (3.8)$$

which is a special case of Kasner solutions. We have a solution

$$\begin{aligned} \mu &= \frac{p \pm \sqrt{pq(p+q-1)}}{p(p+q)}, \\ \nu &= \frac{q \mp \sqrt{pq(p+q-1)}}{q(p+q)}. \end{aligned} \quad (3.9)$$

For $p = 3, q = 7$, we get two future asymptotic solutions:

$$(\mu, \nu) = \text{MF6} \left(\frac{1 + \sqrt{21}}{10}, \frac{7 - 3\sqrt{21}}{70} \right), \quad \text{MF7} \left(\frac{1 - \sqrt{21}}{10}, \frac{7 + 3\sqrt{21}}{70} \right). \quad (3.10)$$

As $t \rightarrow -\infty$, the fourth-order terms dominate. So let us present asymptotic power-law solutions only with quartic terms. Assuming the metric (C.1) with $\epsilon = 1$, our basic Eqs. (2.17) and (2.19) give three algebraic equations. For $p = 3, q = 7$, we have

$$\begin{aligned} &120960\alpha_4\mu\nu^5[7\mu^2 + 7\mu\nu + \nu^2] \\ &- \frac{28\gamma(\mu - \nu)^4}{30375}[5913(\mu - 7)(\mu - 1)^3 - 252(\mu - 1)^2(\nu - 1)(631\mu - 657) \\ &+ 3(\mu - 1)(\nu - 1)^2(50647\mu - 43939) - 2(\nu - 1)^3(29858\mu - 23191)] \\ &- 48\delta(\mu(2\nu + 21) - 7\nu(\mu - 7))[6\mu^2 + 3\mu(7\nu - 1) + 7\nu(4\nu - 1)]^3 = 0, \end{aligned} \quad (3.11)$$

$$\begin{aligned} &40320\alpha_4\nu^5[6\mu^3 + 3\mu^2(8\nu - 7) + 14\mu\nu(\nu - 1) + (\nu - 1)\nu^2] \\ &- \frac{28\gamma(\mu - \nu)^3}{91125}[17739(\mu - 1)^5 - 27(\mu - 1)^4(15259\nu - 12631) + 13334(\nu - 1)^3(\nu^2 - 9\nu + 5) \\ &- 9(\mu - 1)^3(94639\nu^2 - 102980\nu + 20167) + 3(\mu - 1)^2(\nu - 1)(362107\nu^2 - 518030\nu + 162475) \\ &- 2(\mu - 1)(\nu - 1)^2(258865\nu^2 - 468650\nu + 179599)] \\ &- 16\delta(6\mu^2 + 47\mu + 7(\mu + 25)\nu - 28\nu^2 - 168)[6\mu^2 + 3\mu(7\nu - 1) + 7\nu(4\nu - 1)]^3 = 0, \end{aligned} \quad (3.12)$$

$$\begin{aligned} &17280\alpha_4\mu\nu^4[5\mu^2(3\mu - 7) + 2\mu\nu(23\mu - 21) + (38\mu - 7)\nu^2 + 6\nu^3] \\ &+ \frac{4\gamma(\mu - \nu)^3}{30375}[41391(\mu - 1)^5 - 9(\mu - 1)^4(70015\nu - 58189) \\ &- 3(\mu - 1)^3(151793\nu^2 - 58702\nu - 57613) + (\mu - 1)^2(\nu - 1)(784721\nu^2 - 937318\nu + 172253) \\ &- 2(\mu - 1)(\nu - 1)^2(202339\nu^2 - 338869\nu + 106344) - 40002(\nu - 1)^3(2\nu - 1)] \\ &+ 48\delta(\mu(2\mu + 3\nu - 25) - 49\nu + 56)[6\mu^2 + 3\mu(7\nu - 1) + 7\nu(4\nu - 1)]^3 = 0. \end{aligned} \quad (3.13)$$

Two of them are independent as we have shown. Using the values for $\tilde{\alpha}_4$ and $\tilde{\gamma}$ in Eq. (2.26), we have solved these equations numerically and found the following ten solutions for $\delta = 0$:

$$\begin{aligned}
(\mu, \nu) = & \text{MP1}(1.588\,41, 0.319\,25) , & \text{MP2}(0.733\,61, 0.082\,88) , & \text{MP3}(0.722\,46, -0.166\,74) , \\
& \text{MP4}(0.622\,07, -0.400\,40) , & \text{MP5}(0.100\,32, -1.700\,96) , & \text{MP6}(0.022\,04, 0.990\,55) , \\
& \text{MP7}(-0.030\,14, 0.620\,90) , & \text{MP8}(-0.335\,30, 0.850\,24) , & \text{MP9}(-0.668\,48, 0.634\,27) , \\
& \text{MP10}(-0.938\,01, 2.572\,50) . & &
\end{aligned} \tag{3.14}$$

3.2 $\sigma_p = 0, \sigma_q \neq 0$ (or $\sigma_p \neq 0, \sigma_q = 0$)

3.2.1 Generalized de Sitter solutions ($\epsilon = 0$)

Here we have $A_p = \mu^2, A_q = \nu^2 + \tilde{\sigma}_q e^{-2\nu t}, X = \mu^2$ and $Y = \nu^2$, where $\tilde{\sigma}_q \equiv \sigma_q/b_0^2$. It is easy to see that there is no exact solution unless $\nu = 0$, in which case we have constant $A_p = X = \mu^2, A_q = \tilde{\sigma}_q$ and $Y = 0$. Our basic Eqs. (2.17) and (2.19) now read

$$\begin{aligned}
& \alpha_1 [p_1 \mu^2 + q_1 \tilde{\sigma}_q] + \alpha_4 [p_7 \mu^8 + 4p_5 q_1 \mu^6 \tilde{\sigma}_q + 6p_3 q_3 \mu^4 \tilde{\sigma}_q + 4p_1 q_5 \mu^2 \tilde{\sigma}_q^3 + q_7 \tilde{\sigma}_q^4] \\
& + \frac{\gamma p q_1 N_1(q, p)}{(D-1)^3 (D-2)^4} [(p-7)\mu^2 + (p+1)\tilde{\sigma}_q] [\mu^2 + \tilde{\sigma}_q]^3 \\
& + \delta [p(p-7)\mu^2 + q_1 \tilde{\sigma}_q] [p_1 \mu^2 + q_1 \tilde{\sigma}_q]^3 = 0 ,
\end{aligned} \tag{3.15}$$

$$\begin{aligned}
& \alpha_1 [(p+1)_0 \mu^2 + (q-1)_2 \tilde{\sigma}_q] \\
& + \alpha_4 [(p+1)_6 \mu^8 + 4(p+1)_4 q_2 \mu^6 \tilde{\sigma}_q + 6(p+1)_2 q_4 \mu^4 \tilde{\sigma}_q^2 + 4(p+1)_0 q_6 \mu^2 \tilde{\sigma}_q^3 + q_8 \tilde{\sigma}_q^4] \\
& + \frac{\gamma(q-1)(p+1)_0 N_1(q, p)}{(D-1)^3 (D-2)^4} [q\mu^2 + (q-8)\tilde{\sigma}_q] [\mu^2 + \tilde{\sigma}_q]^3 \\
& + \delta [(p+1)_0 \mu^2 + (q-1)(q-8)\tilde{\sigma}_q] [(p+1)_0 \mu^2 + q_1 \tilde{\sigma}_q]^3 = 0 .
\end{aligned} \tag{3.16}$$

We note that Eq. (2.18) gives the same equation as Eq. (2.19) for $\nu = 0$ and need not be taken into account. Setting $\delta = 0$, it turns out that there is no solution for $\sigma_p = 0, \sigma_q \neq 0$. For the case of $\sigma_p \neq 0$ and $\sigma_q = 0$, exchanging μ, p and ν, q , we find that there is also no solution. Although there is no this kind of solution for $\delta = 0$, we find solutions for $\delta \neq 0$ as discussed in § 4.2.1.

3.2.2 Power-law solutions ($\epsilon = 1$)

Here we have $A_p = \mu^2, A_q = \nu^2 + \tilde{\sigma}_q e^{2(1-\nu)t}, X = \mu(\mu-1)$ and $Y = \nu(\nu-1)$. We have only asymptotic solutions in most cases.

(1) $\nu > 1$:

For $t \rightarrow \infty$, the EH term dominates and $A_q \rightarrow \nu^2$. The solutions are the same as $\sigma_p = \sigma_q = 0$ case in section 3.1.2. However, there is no solution with $\nu > 1$.

For $t \rightarrow -\infty, A_q \rightarrow \tilde{\sigma}_q e^{2(1-\nu)t}$ and there is no solution.

(2) $\nu < 1$:

For $t \rightarrow \infty, A_q \rightarrow \tilde{\sigma}_q e^{2(1-\nu)t}$ and there is no solution.

For $t \rightarrow -\infty, A_q \rightarrow \nu^2$ and the solutions are the same as $\sigma_p = \sigma_q = 0$ case in section 3.1.2. Choosing solutions with $\nu < 1$ from Eq. (3.14), we get nine solutions MP1 – MP9.

(3) $\nu = 1$:

We have $A_p = \mu^2, A_q = 1 + \tilde{\sigma}_q, X = \mu(\mu-1)$ and $Y = 0$.

For $t \rightarrow \infty$, the EH term dominates and the solution is

$$(\mu, \nu, \tilde{\sigma}_q) = \text{ME12}(0, 1, -1) . \quad (3.17)$$

Actually this is an exact solution.

For $t \rightarrow -\infty$, fourth-order terms dominate. Our basic independent Eqs. (2.17) and (2.19) give

$$\begin{aligned} & \alpha_4 [p_7 \mu^8 + 8p_6 q \mu^7 + 4p_5 q_1 \mu^6 (6 + A_q) + 8p_4 q_2 \mu^5 (4 + 3A_q) + 2p_3 q_3 \mu^4 (8 + 24A_q + 3A_q^2) \\ & + 8p_2 q_4 \mu^3 A_q (4 + 3A_q) + 4p_1 q_5 \mu^2 A_q^2 (6 + A_q) + 8p q_6 \mu A_q^3 + q_7 A_q^4] \\ & + \frac{\gamma p q_1 N_1(q, p)}{(D-1)^3 (D-2)^4} [(p-7)\mu^2 + 2(3p+4q-21)\mu - 8(p+q-6) + (p+1)A_q] [\mu(\mu-2) + A_q]^3 \\ & + \delta [p(p-7)\mu^2 + 2p(q-21)\mu - 48q + q_1 A_q] [p(p+1)\mu^2 + 2p(q-1)\mu + q_1 A_q]^3 = 0 , \end{aligned} \quad (3.18)$$

$$\begin{aligned} & \alpha_4 [(p+1)_6 \mu^8 + 8p_5 (pq - 2p + 6) \mu^7 + 4p_4 (q-1) \mu^6 (6(pq - 4p - q + 12) + (p+1)(q-2)A_q) \\ & + 8p_3 (q-1)_2 \mu^5 (4p(q-6) - 8(q-9) + 3(pq - 4p + 4)A_q) \\ & + 2p_2 (q-1)_3 \mu^4 (8(p-3)(q-8) + 24(pq - 6p - q + 10)A_q + 3(p+1)(q-4)A_q^2) \\ & + 8p_1 (q-1)_4 \mu^3 A_q (4(p-2)(q-8) + 3(pq - 6p + 2)A_q) \\ & + 4p(q-1)_5 \mu^2 A_q^2 (6(p-1)(q-8) + (p+1)(q-6)A_q) + 8p(q-8)(q-1)_6 \mu A_q^3 + (q-1)_8 A_q^4] \\ & + \frac{\gamma p (q-1) N_1(q, p)}{(D-1)^3 (D-2)^4} [\mu(\mu-2) + A_q]^3 [(p+1)q\mu^2 - 2(4p^2 + 5pq - 28p + q)\mu \\ & - 8(q-8)(p+q-6) + (p+1)(q-8)A_q] \\ & + \delta [(p+1)_0 \mu^2 + 2p(q-29)\mu + (q-8)((q-1)A_q - 48)] [(p+1)_0 \mu^2 + 2p(q-1)\mu + q_1 A_q]^3 = 0 . \end{aligned} \quad (3.19)$$

For $p = 3$ and $q = 7$, we find the solutions ME10 in Eq. (3.17) and

$$\begin{aligned} (\mu, \nu, \tilde{\sigma}_q) = & \text{MP11}(6.679\,58, 1 - 2.106\,62) , & \text{MP12}(6.085\,83, 1, -1.357\,93) , & \text{MP13}(2, 1, -1) , \\ & \text{MP14}(0.481\,80, 1, -1.238\,02) , & \text{MP15}(0.072\,89, 1, -2.208\,58) . \end{aligned} \quad (3.20)$$

For the case of $\sigma_p \neq 0$ and $\sigma_q = 0$, exchanging μ, p and ν, q , we obtain one exact solution ME13 and thirteen asymptotic solutions MP2 – MP10 in Eq. (3.14) and the following solutions MP16 – MP19:

$$(\mu, \nu, \tilde{\sigma}_p) = \text{ME13}(1, 0, -1) , \quad (3.21)$$

$$\begin{aligned} (\mu, \nu, \tilde{\sigma}_p) = & \text{MP16}(1, 0.268\,18, 1.438\,12) , \text{MP17}(1, -9.177\,79, 4.633\,79) , \\ & \text{MP18}(1, 0.931\,76, -3.965\,11) , \text{MP19}(1, -0.122\,50, -1.414\,71) , \end{aligned} \quad (3.22)$$

where $\tilde{\sigma}_p \equiv \sigma_p/a_0^2$.

3.3 $\sigma_p \sigma_q \neq 0$

3.3.1 Generalized de Sitter solutions ($\epsilon = 0$)

If $\mu = \nu = 0$, our basic Eqs. (2.17) and (2.18) reduce to

$$\begin{aligned} & \alpha_1 [p_1 \tilde{\sigma}_p + q_1 \tilde{\sigma}_q] + \alpha_4 [p_7 \tilde{\sigma}_p^4 + 4p_5 q_1 \tilde{\sigma}_p^3 \tilde{\sigma}_q + 6p_3 q_3 \tilde{\sigma}_p^2 \tilde{\sigma}_q^2 + 4p_1 q_5 \tilde{\sigma}_p \tilde{\sigma}_q^3 + q_7 \tilde{\sigma}_q^4] \\ & + \frac{\gamma}{(D-1)^3 (D-2)^4} [p_1 (q+1)_0 N_1(p, q) \tilde{\sigma}_p^4 + 4p_1 q_1 N_1(p, q) \tilde{\sigma}_p^3 \tilde{\sigma}_q \\ & + 2p_1 q_1 N_2(p, q) \tilde{\sigma}_p^2 \tilde{\sigma}_q^2 + 4p_1 q_1 N_1(q, p) \tilde{\sigma}_p \tilde{\sigma}_q^3 + (p+1)_0 q_1 N_1(q, p) \tilde{\sigma}_q^4] \\ & + \delta [p_1 \tilde{\sigma}_p + q_1 \tilde{\sigma}_q]^4 = 0 , \end{aligned} \quad (3.23)$$

$$\alpha_1 [(p-1)_2 \tilde{\sigma}_p + q_1 \tilde{\sigma}_q]$$

$$\begin{aligned}
& + \alpha_4 [(p-1)_8 \tilde{\sigma}_p^4 + 4(p-1)_6 q_1 \tilde{\sigma}_p^3 \tilde{\sigma}_q + 6(p-1)_4 q_3 \tilde{\sigma}_p^2 \tilde{\sigma}_q^2 + 4(p-1)_2 q_5 \tilde{\sigma}_p \tilde{\sigma}_q^3 + q_7 \tilde{\sigma}_q^4] \\
& + \frac{\gamma}{(D-1)^3 (D-2)^4} [(p-8)(p-1)(q+1)_0 N_1(p, q) \tilde{\sigma}_p^4 + 4(p-6)(p-1) q_1 N_1(p, q) \tilde{\sigma}_p^3 \tilde{\sigma}_q \\
& + 2(p-4)(p-1) q_1 N_2(p, q) \tilde{\sigma}_p^2 \tilde{\sigma}_q^2 + 4(p-2)(p-1) q_1 N_1(q, p) \tilde{\sigma}_p \tilde{\sigma}_q^3 + (p+1)_0 q_1 N_1(q, p) \tilde{\sigma}_q^4] \\
& + \delta [(p-8)(p-1) \tilde{\sigma}_p + q_1 \tilde{\sigma}_q] (p_1 \tilde{\sigma}_p + q_1 \tilde{\sigma}_q)^3 = 0, \tag{3.24}
\end{aligned}$$

$$\begin{aligned}
& \alpha_1 [p_1 \tilde{\sigma}_p + (q-1)_2 \tilde{\sigma}_q] \\
& + \alpha_4 [p_7 \tilde{\sigma}_p^4 + 4p_5 (q-1)_2 \tilde{\sigma}_p^3 \tilde{\sigma}_q + 6p_3 (q-1)_4 \tilde{\sigma}_p^2 \tilde{\sigma}_q^2 + 4p_1 (q-1)_6 \tilde{\sigma}_p \tilde{\sigma}_q^3 + (q-1)_8 \tilde{\sigma}_q^4] \\
& + \frac{\gamma}{(D-1)^3 (D-2)^4} [p_1 (q+1)_0 N_1(p, q) \tilde{\sigma}_p^4 + 4p_1 (q-1)_2 N_1(p, q) \tilde{\sigma}_p^3 \tilde{\sigma}_q \\
& + 2p_1 (q-4)(q-1) N_2(p, q) \tilde{\sigma}_p^2 \tilde{\sigma}_q^2 + 4p_1 (q-6)(q-1) N_1(q, p) \tilde{\sigma}_p \tilde{\sigma}_q^3 + (p+1)_0 (q-8)(q-1) N_1(q, p) \tilde{\sigma}_q^4] \\
& + \delta [(p_1 \tilde{\sigma}_p + (q-8)(q-1) \tilde{\sigma}_q) (p_1 \tilde{\sigma}_p + q_1 \tilde{\sigma}_q)^3] = 0, \tag{3.25}
\end{aligned}$$

where $N_2(p, q)$ is defined by

$$\begin{aligned}
N_2(p, q) = & p^5(9q-16) + p^4(3q^2-1) + p^3(14q^2-21q+26) + 3p^2(q-3) - 12p^3q^3 \\
& - 13p^2q^2 + 9pq + 3(p-3)q^2 + (14p-21p+26)q^3 + (3p^2-1)q^4 + (9p-16)q^5. \tag{3.26}
\end{aligned}$$

For $p = 3$, $q = 7$ and $\delta = 0$, we find that there is no solution.

If $\mu \neq 0$ and $\nu = 0$, it is clear that there is no exact solution. For asymptotic solutions, we can search for them by setting $A_p = \mu^2$, $A_q = \tilde{\sigma}_q$, $X = \mu^2$ and $Y = 0$. This is the same condition in § 3.2.1, and we have no solution. The case with $\mu = 0$ and $\nu \neq 0$ is obtained by exchanging p , μ and q , ν . We find again that there is no solution.

For $\mu\nu \neq 0$, if our ansatz for solution is imposed, it is easy to see that there is no solution if μ and ν are of the opposite signs. If they are of the same sign, either $t \rightarrow +\infty$ or $t \rightarrow -\infty$ gives $A_p \rightarrow \mu^2$, $A_q \rightarrow \nu^2$ and there may be solutions. For $\delta = 0$, however, we see that Eq. (3.4) gives no solution of the same sign.

3.3.2 Power-law solutions ($\epsilon = 1$)

In this case, we first consider the cases when both μ and ν are not equal to 1.

(1) $\mu > 1$ and $\nu > 1$:

For $t \rightarrow \infty$, the EH term dominates and we obtain the asymptotic solutions in § 3.1.2. Again no solutions satisfy the condition of $\mu > 1$ and $\nu > 1$ (see Eq. (3.8)) and hence there is no asymptotic solution of our form.

As $t \rightarrow -\infty$, the fourth-order terms become dominant and we find no consistent solution from the forth-order terms.

(2) $\mu < 1$ and $\nu < 1$:

As $t \rightarrow \infty$ with EH dominance, we again find no consistent solution. As $t \rightarrow -\infty$ with fourth-order-term dominance, we obtain eight asymptotic solutions MP2 – MP9 from Eq. (3.14) in § 3.1.2:

(3) $\mu > 1$ and $\nu < 1$:

As $t \rightarrow \infty$, $A_p \rightarrow \mu^2$ and $A_q \rightarrow \tilde{\sigma}_q e^{2(1-\nu)t}$. This is similar to the case (2) in § 3.2.2 and there is no solution of our form.

As $t \rightarrow -\infty$, $A_p \rightarrow \tilde{\sigma}_p e^{2(1-\mu)t}$ and $A_q \rightarrow \nu^2$. We find no solution.

(4) $\mu < 1$ and $\nu > 1$:

Here we reach the same result by exchanging p, μ and q, ν . No asymptotic solution of our form is obtained for both $t \rightarrow \pm\infty$.

Next, we discuss the cases in which one of μ or ν is equal to 1 and the other is not:

(5) $\mu > 1$ and $\nu = 1$:

As $t \rightarrow \infty$ with EH dominance, $A_p \rightarrow \mu^2$, and we recover the case of $\sigma_p = 0, \sigma_q \neq 0$. However, there is no solution with $\mu > 1$. We do not have any asymptotic solution of our form. As $t \rightarrow -\infty$ with fourth-order-term dominance, $A_p \rightarrow \tilde{\sigma}_p e^{2(1-\mu)t}$. We again do not have any asymptotic solution of our form.

(6) $\mu < 1$ and $\nu = 1$:

As $t \rightarrow \infty$ with EH dominance, A_p diverges as $\tilde{\sigma}_p e^{2(1-\mu)t}$. There is no asymptotic solution of our form. As $t \rightarrow -\infty$, we again recover the case of $\sigma_p = 0, \sigma_q \neq 0$ with the fourth-order-term dominance and solutions in (3.17) and (3.20). (Note that (3.17) was an exact solution for $\sigma_p = 0$.) Choosing those with $\mu < 1$, we get asymptotic power-law solutions

$$(\mu, \nu, \tilde{\sigma}_q) = \text{MP23}(0, 1, -1) . \quad (3.27)$$

form Eq. (3.17), and MP14 and MP15 from Eq. (3.20).

(7) $\mu = 1$ and $\nu > 1$:

The analysis is almost the same as the case (5). There is no asymptotic solution.

(8) $\mu = 1$ and $\nu < 1$:

The analysis is almost the same as the case (6), and we find the asymptotic solutions

$$(\mu, \nu, \tilde{\sigma}_q) = \text{MP24}(1, 0, -1) . \quad (3.28)$$

form Eq. (3.21), and MP16 – MP19 for $t \rightarrow -\infty$, which are the same as the case of $\sigma_p \neq 0, \sigma_q = 0$ given in Eq. (3.22).

Finally, we consider the remaining case.

(9) $\mu = 1$ and $\nu = 1$:

Here we have constant $A_p = 1 + \tilde{\sigma}_p$ and $A_q = 1 + \tilde{\sigma}_q$. As $t \rightarrow +\infty$, the EH term is dominant, and we have

$$\begin{aligned} p_1 A_p + q_1 A_q + 2pq &= 0, \\ (p-1)_2 A_p + q_1 A_q + 2(p-1)q &= 0, \\ p_1 A_p + (q-1)_2 A_q + 2p(q-1) &= 0. \end{aligned} \quad (3.29)$$

The solution is given by

$$A_p = -\frac{q}{p-1}, \quad A_q = -\frac{p}{q-1}. \quad (3.30)$$

This is the solution found in Ref. [11] which exhibits eternal accelerating expansion when higher order effects are taken into account. For $p = 3, q = 7$, we get the following future asymptotic solution:

$$(\mu, \nu, \tilde{\sigma}_p, \tilde{\sigma}_q) = \text{MF8} \left(1, 1, -\frac{9}{2}, -\frac{3}{2} \right) . \quad (3.31)$$

For $t \rightarrow -\infty$ with fourth-order-term dominance, we get two independent equations for $\tilde{\sigma}_p$ and $\tilde{\sigma}_q$ from

Eqs. (2.17) and (2.18).

$$\begin{aligned}
& \alpha_4 [p_7(1 + \tilde{\sigma}_p)^4 + 4(1 + \tilde{\sigma}_p)^3(2qp_6 + p_5q_1(1 + \tilde{\sigma}_q)) + 24(1 + \tilde{\sigma}_p)^2(p_5q_1 + p_4q_2(1 + \tilde{\sigma}_q)) \\
& + 32p_4q_2(1 + \tilde{\sigma}_p) + 32p_2q_4(1 + \tilde{\sigma}_q) + 24(p_1q_5 + p_2q_4(1 + \tilde{\sigma}_p))(1 + \tilde{\sigma}_q)^2 \\
& + 4(2pq_6 + p_1q_5(1 + \tilde{\sigma}_p))(1 + \tilde{\sigma}_q)^3 + q_7(1 + \tilde{\sigma}_q)^4 + 6p_3q_3(1 + \tilde{\sigma}_p)^2(1 + \tilde{\sigma}_q)^2 \\
& + 48p_3q_3(1 + \tilde{\sigma}_p)(1 + \tilde{\sigma}_q) + 16p_3q_3] \\
& + \frac{\gamma}{(D-1)^3(D-2)^4} [p_1(q+1)_0N_1(p, q)\tilde{\sigma}_p^4 + 4p_1q_1N_1(p, q)\tilde{\sigma}_p^3\tilde{\sigma}_q + 2p_1q_1N_2(p, q)\tilde{\sigma}_p^2\tilde{\sigma}_q^2 \\
& + 4p_1q_1N_1(q, p)\tilde{\sigma}_p\tilde{\sigma}_q^3 + (p+1)_0q_1N_1(q, p)\tilde{\sigma}_q^4] \\
& + \delta[(p+q-49)(p+q) + p_1\tilde{\sigma}_p + q_1\tilde{\sigma}_q)((p+q-1)(p+q) + p_1\tilde{\sigma}_p + q_1\tilde{\sigma}_q)^3] = 0, \tag{3.32}
\end{aligned}$$

$$\begin{aligned}
& \alpha_4 [(p-1)_8(1 + \tilde{\sigma}_p)^4 + 4(1 + \tilde{\sigma}_p)^3(2q(p-1)_7 + (p-1)_6q_1(1 + \tilde{\sigma}_q)) \\
& + 32(p-1)_5q_2(1 + \tilde{\sigma}_p) + 32(p-1)_3q_4(1 + \tilde{\sigma}_q) + 24((p-1)_2q_5 + (p-1)_3q_4(1 + \tilde{\sigma}_p))(1 + \tilde{\sigma}_q)^2 \\
& + 4(2(p-1)q_6 + (p-1)_2q_5(1 + \tilde{\sigma}_p))(1 + \tilde{\sigma}_q)^3 + q_7(1 + \tilde{\sigma}_q)^4 + 6(p-1)_4q_3(1 + \tilde{\sigma}_p)^2(1 + \tilde{\sigma}_q)^2 \\
& + 48(p-1)_4q_3(1 + \tilde{\sigma}_p)(1 + \tilde{\sigma}_q) + 16(p-1)_4q_3] \\
& + \frac{\gamma}{(D-1)^3(D-2)^4} [-(p-1)N_1(p, q)\tilde{\sigma}_p^3\{(p-8)(q+1)_0\tilde{\sigma}_p + 4(p-6)q_1\tilde{\sigma}_q - 8(p+q-7)^2\} \\
& - q_1N_1(q, p)\tilde{\sigma}_q^3\{4(p-1)_2\tilde{\sigma}_p + (p+1)_0\tilde{\sigma}_q + 8(p+q-7)^2\} \\
& + 4(p-1)q_1(p+q-7)^2\tilde{\sigma}_p\tilde{\sigma}_q(N_3(p, q)\tilde{\sigma}_p - N_3(q, p)\tilde{\sigma}_q) - 2(p-1)(p-4)N_2(p, q)\tilde{\sigma}_p^2\tilde{\sigma}_q^2] \\
& + \delta[((p+q-49)(p+q-8) + (p-8)(p-1)\tilde{\sigma}_p + q_1\tilde{\sigma}_q)((p+q-1)(p+q) + p_1\tilde{\sigma}_p + q_1\tilde{\sigma}_q)^3] = 0, \tag{3.33}
\end{aligned}$$

where we define $N_3(p, q)$ as

$$\begin{aligned}
N_3(p, q) &= pp_1(7p^2 + 35p - 18) - 3p_1(6p^2 - p - 6)q - (32p^3 - 49p^2 + 33p - 18)q^2 \\
&+ (16p^2 + 15p - 47)q^3 + (21p - 41)q^4 - 2q^5. \tag{3.34}
\end{aligned}$$

For $p = 3$, $q = 7$ and $\delta = 0$, we find four solutions

$$\begin{aligned}
(\mu, \nu, \tilde{\sigma}_p, \tilde{\sigma}_q) &= \text{MP25}(1, 1, 2.370\,16, -1.262\,93), & \text{MP26}(1, 1, -1.412\,83, -120.598\,08), \\
&\text{MP27}(1, 1, -4.376\,02, -0.120\,21), & \text{MP28}(1, 1, -5.762\,67, -2.588\,03). \tag{3.35}
\end{aligned}$$

We summarize our results obtained here in Appendix D. The exact solutions for $\delta = 0$ are listed in Table 5, future asymptotic solutions in Table 6 and past asymptotic solutions in Table 7. The numbering of solutions are given for those with $\delta \neq 0$ (see next section). For example, in Table 5, we do not find solutions ME2 $_{\pm}$ – ME11 $_{\pm}$.

4 Solutions in M theory for $\delta \neq 0$

In this section, we search for exact generalized de Sitter solutions for $-10 \leq \delta \leq 10$ whereas we give a few examples for future and past asymptotic solutions for particular δ to avoid vexatious complications. As typical examples, we mainly focus on the case of $\delta = -0.001$ and $\delta = -0.1$ as to future and past asymptotic solutions in § 4.1.2, 4.2.2 and 4.3.1 – 4.3.2, but we also study how the results change depending on δ .

4.1 $\sigma_p = \sigma_q = 0$

4.1.1 Generalized de Sitter Solutions ($\epsilon = 0$)

In this case, we have the same Eqs. (3.1) and (3.2) with a free parameter $\delta \neq 0$. For the given value of δ , we may solve these equations numerically. In Figs. 1 and 2, we depict numerical solutions ME i_+ (δ, μ_i, ν_i) ($i = 1, \dots, 5$)

with $\mu_i \geq 0$ when δ is varied. We note that there are always time-reversed solutions $\text{ME}i_-(\delta, \mu'_i, \nu'_i)$ ($i = 1, \dots, 5$) obtained by $(\mu'_i, \nu'_i) = (-\mu_i, -\nu_i)$ which are not shown explicitly. We find five solutions $\text{ME}1_+ - \text{ME}5_+$ for $\delta < 0$, while just one $\text{ME}1_+$ for $\delta > 0$. For the case of $\delta = 0$, we have a solution $\text{ME}1_+$ which is consistent with the result of § 3.1.1. In Table 1, we summarize their properties.

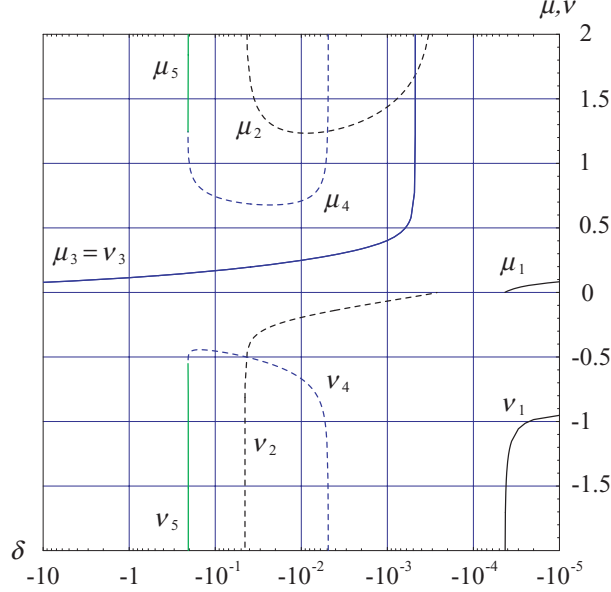


Figure 1: Five Generalized de Sitter solutions with $\sigma_p = \sigma_q = 0$ with respect to $\delta < 0$.

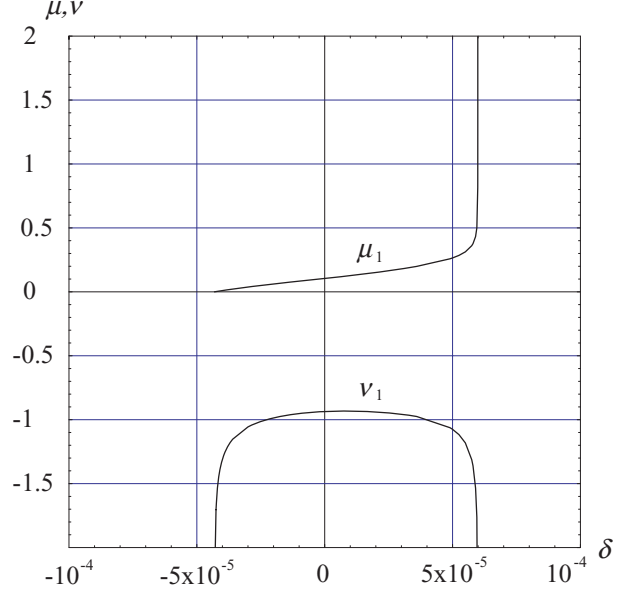


Figure 2: One Generalized de Sitter solution with $\sigma_p = \sigma_q = 0$ near the origin of δ .

Each pair of points (δ, μ_i, ν_i) with the same value of δ gives one solution. Number of solutions changes with the value of δ . There is another set of time-reversed solutions $\text{ME}i_-$ with $\mu_i < 0$. No solution exists for $10^{-4} < \delta$.

Table 1: Generalized de Sitter Solutions $\text{ME}i_+$ ($i = 1, \dots, 5$) with $\mu_i \geq 0$ for various values of δ . Five eigenmodes for linear perturbations are also shown. (ms, nu) means that there are m stable modes and n unstable modes. The solution has many stable modes if its 10-volume expansion rate $3\mu + 7\nu$ is positive.

Solution	Property	Range	Stability	$3\mu_i + 7\nu_i$
$\text{ME}1_+$	$\nu_1 < 0 < \mu_1$	$-0.000\,043\,11 < \delta < 0$	(0s,5u)	—
		$\delta = 0$	(1s,2u)	—
		$0 < \delta < 0.000\,059\,88$	(1s,4u)	—
$\text{ME}2_+$	$\nu_2 < 0 < \mu_2$	$-0.045\,20 < \delta < -0.002\,649$	(4s,1u)	+
$\text{ME}3_+$	$0 < \mu_3 = \nu_3$	$\delta < -0.000\,4732$	(3s,0u)	+
$\text{ME}4_+$	$\nu_4 < 0 < \mu_4$	$-0.2073 < \delta < -0.004\,852$	(1s,4u)	—
$\text{ME}5_+$	$\nu_5 < 0 < \mu_5$	$-0.2073 < \delta < -0.2056$	(2s,3u)	—

We find that there are two solutions around the value $\delta \sim -0.001$ in Fig. 1. Especially, we have the following solutions for $\delta = -0.001$:

$$(\delta, \mu, \nu) = \text{ME}2_+(-0.001, 1.437\,87, -0.067\,0662), \quad \text{ME}3_+(-0.001, 0.402\,934, 0.402\,934). \quad (4.1)$$

It is interesting to see how the solutions change for other value of δ . For instance, we have the following two solutions for $\delta = -0.1$:

$$(\delta, \mu, \nu) = \text{ME}3_+(-0.1, 0.168\,203, 0.168\,203), \quad \text{ME}4_+(-0.1, 0.742\,918, -0.453\,997). \quad (4.2)$$

where we use the same names for the solutions connected when δ is changed. The value of μ_i and ν_i of the solution with the same sign change with the value of δ as in Eqs. (4.1) and (4.2). We will also study a linear perturbation around these solutions in § 5.1.

4.1.2 Power-law solutions ($\epsilon = 1$)

There is no exact solutions, but we have asymptotic solutions for various values of δ . We will give them explicitly for $\delta = -0.001$ and $\delta = -0.1$.

As $t \rightarrow \infty$ with EH dominance, we get the same solutions MF6 and MF7 given by Eq. (3.10) in § 3.1.2

As $t \rightarrow -\infty$, the forth order terms dominate. We have the same Eqs. (3.11) – (3.13) in § 3.1.2 with $\delta \neq 0$. For $\delta = -0.001$, we find the following twelve solutions

$$\begin{aligned}
(\delta, \mu, \nu) = & \text{MP6}(-0.001, 121.218, -5.48783) , & \text{MP7}(-0.001, 27.0789, 27.0789) , \\
& \text{MP8}(-0.001, 26.6578, -37.1453) , & \text{MP9}(-0.001, 2.61038, -0.118736) , \\
& \text{MP10}(-0.001, 0.737553, -0.0863059) , & \text{MP11}(-0.001, 0.726753, -0.15141) , \\
& \text{MP12}(-0.001, 0.190928, 0.139524) , & \text{MP13}(-0.001, 0.154834, 0.154834) , \\
& \text{MP14}(-0.001, 0.120104, 0.169926) , & \text{MP15}(-0.001, -0.757551, 0.625032) , \\
& \text{MP16}(-0.001, -1.16161, 1.49783) , & \text{MP17}(-0.001, -2.40756, 0.598625) ,
\end{aligned} \tag{4.3}$$

while the following eight for $\delta = -0.1$

$$\begin{aligned}
(\delta, \mu, \nu) = & \text{MP6}(-0.1, 14.0692, 14.0692) , & \text{MP7}(-0.1, 8.24022, -2.53152) , \\
& \text{MP8}(-0.1, 0.229099, 0.151155) , & \text{MP9}(-0.1, 0.174972, 0.174972) , \\
& \text{MP10}(-0.1, 0.123489, 0.196532) , & \text{MP11}(-0.1, -5.57678, 3.39005) , \\
& \text{MP12}(-0.1, -60.445, 36.7782) , & \text{MP13}(-0.1, -225.859, 69.5863) .
\end{aligned} \tag{4.4}$$

4.2 $\sigma_p = 0, \sigma_q \neq 0$ (or $\sigma_p \neq 0, \sigma_q = 0$)

4.2.1 Generalized de Sitter solutions ($\epsilon = 0$)

Here we have $A_p = \mu^2$, $A_q = \nu^2 + \tilde{\sigma}_q e^{-2\nu t}$, $X = \mu^2$ and $Y = \nu^2$. It is easy to see that there is no exact solution unless $\nu = 0$, in which case we have constant $A_p = X = \mu^2$, $A_q = \tilde{\sigma}_q$ and $Y = 0$. Our basic equations reduce to Eqs. (3.15) and (3.16). In Figs. 3 and 4, we depict numerical solutions $\text{ME}i_+(\delta, \mu_i, \nu_i = 0, \tilde{\sigma}_q)$ ($i = 6, 7$) with $\mu_i > 0$ as a function of δ . There are also time-reversed solutions $\text{ME}i_-(\delta, \mu'_i, \nu_i = 0, \tilde{\sigma}_q)$ ($i = 6, 7$) obtained by $\mu'_i = -\mu_i$ which are not shown. We find a solution $\text{ME}6_+$ for $\delta < 0$, while $\text{ME}7_+$ for $\delta < 0$. In the vicinity of $\delta = 0$, we find no solution as discussed in § 3.2.1. In the anterior part of Table 2, we summarize their properties.

Table 2: Generalized de Sitter Solutions $\text{ME}i_+(\delta, \mu_i, \nu_i)$ ($i = 6, \dots, 11$) with $\mu_i, \nu_i \geq 0$ for various values of δ . Six eigenmodes for linear perturbations are also shown. (ms, nu) means that there are m stable modes and n unstable modes.

Solution	Property	Range	Stability	$3\mu_i + 7\nu_i$
$\text{ME}6_+$	$\nu_6 = 0, 0 < \mu_6, \tilde{\sigma}_q(6)$	$\delta < -0.0005589$	(5s,1u)	+
$\text{ME}7_+$	$\nu_7 = 0, \tilde{\sigma}_q(7) < 0 < \mu_6$	$0.002999 < \delta$	(4s,2u)	+
$\text{ME}8_+$	$\mu_8 = 0, 0 < \tilde{\sigma}_p(8), \nu_8$	$-0.003163 < \delta < -0.0005650$	(4s,2u)	+
$\text{ME}9_+$	$\mu_9 = 0, 0 < \tilde{\sigma}_p(9), \nu_9$	$\delta < -0.0005657$	(5s,1u)	+
$\text{ME}10_+$	$\mu_{10} = 0, \tilde{\sigma}_p(10) < 0 < \nu_{10}$	$\delta < -0.00004349$	(5s,1u)	+
$\text{ME}11_+$	$\mu_{11} = 0, \tilde{\sigma}_p(11) < 0 < \nu_{11}$	$-0.08522 < \delta < -0.003164$	(4s,2u)	+

Especially, we find the following solution for $\delta = -0.001$:

$$(\delta, \mu, \nu, \tilde{\sigma}_q) = \text{ME}6_+(-0.001, 0.775074, 0, 0.278981) . \tag{4.5}$$

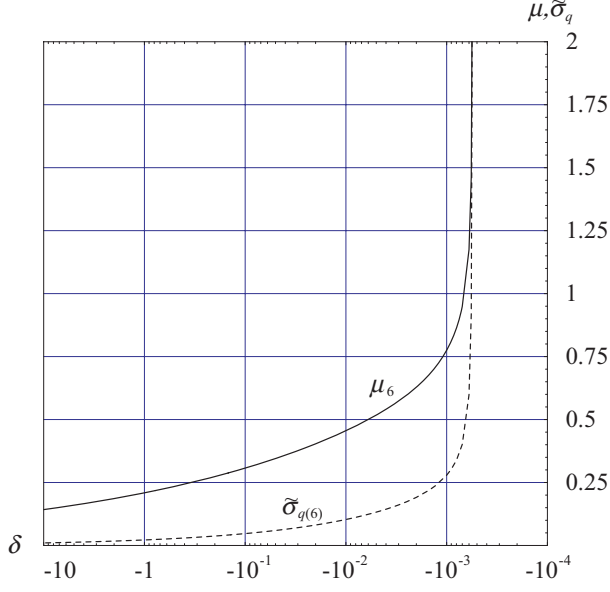


Figure 3: One generalized de Sitter solution with $\sigma_p = 0$, $\sigma_q \neq 0$ for $\delta < 0$.

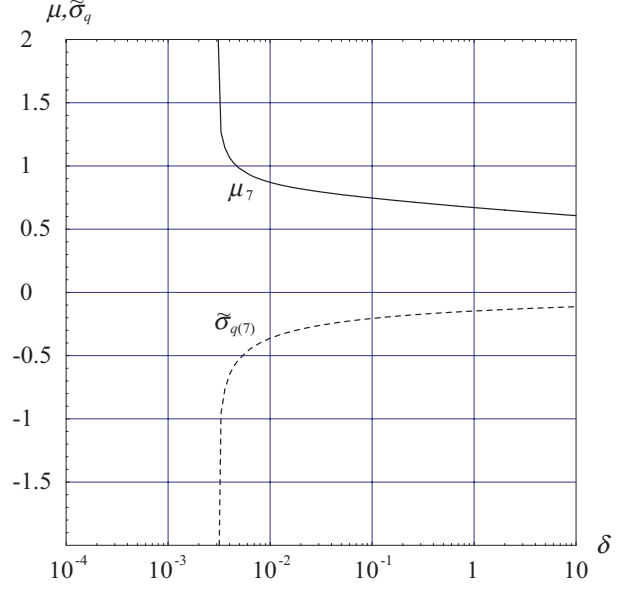


Figure 4: One Generalized de Sitter solution with $\sigma_p = 0$, $\sigma_q \neq 0$ for $0 < \delta$.

while the following for $\delta = -0.1$

$$(\delta, \mu, \nu, \tilde{\sigma}_q) = \text{ME6}_+(-0.1, 0.307198, 0, 0.0471560), \quad (4.6)$$

For the case of $\sigma_p \neq 0$ and $\sigma_q = 0$, exchanging μ, p and ν, q , we find four solutions $\text{ME8}_+ - \text{ME11}_+$. In the vicinity of $\delta = 0$, we find again no solution as discussed in § 3.2.1. In Fig. 5, we depict these four numerical solutions with $\nu_i > 0$ with respect to δ . There are also time-reversed solutions $\text{ME}i_-(\delta, \mu_i = 0, \nu'_i, \tilde{\sigma}_p)$ ($i = 6, 7$) obtained by $\nu'_i = -\nu_i$ which are not shown. In the posterior part of Table 2, we summarize their properties.

Especially, for $\delta = -0.001$ and $\delta = -0.1$, we have the following solutions, respectively:

$$\begin{aligned} (\delta, \mu, \nu, \tilde{\sigma}_p) = \text{ME8}_+(-0.001, 0, 0.491829, 0.911910), \quad \text{ME9}_+(-0.001, 0, 0.401567, 1.97026), \\ \text{ME10}_+(-0.001, 0, 0.408387, -0.672260). \end{aligned} \quad (4.7)$$

$$\text{ME9}_+(-0.1, 0, 0.201057, 0.141557), \quad \text{ME10}_+(-0.1, 0, 0.314065, -0.757016). \quad (4.8)$$

4.2.2 Power-law solutions ($\epsilon = 1$)

Here we have $A_p = \mu^2, A_q = \nu^2 + \tilde{\sigma}_q e^{2(1-\nu)t}, X = \mu(\mu - 1)$ and $Y = \nu(\nu - 1)$. We have only asymptotic solutions in most cases.

(1) $\nu > 1$:

In this case, we have same result in § 3.2.2 (1), and there is no asymptotic solution of our form.

(2) $\nu < 1$:

For $t \rightarrow \infty$, $A_q \rightarrow \tilde{\sigma}_q e^{2(1-\nu)t}$ and there is no solution.

For $t \rightarrow -\infty$, $A_q \rightarrow \nu^2$ and the solutions are the same as $\sigma_p = \sigma_q = 0$ case in § 4.1.2. From Eqs. (4.3) and (4.4), we get ten past asymptotic solutions MP6, MP8 – MP15 and MP17 for $\delta = -0.001$, while the four solutions MP7 – MP10 for $\delta = -0.1$.

(3) $\nu = 1$:

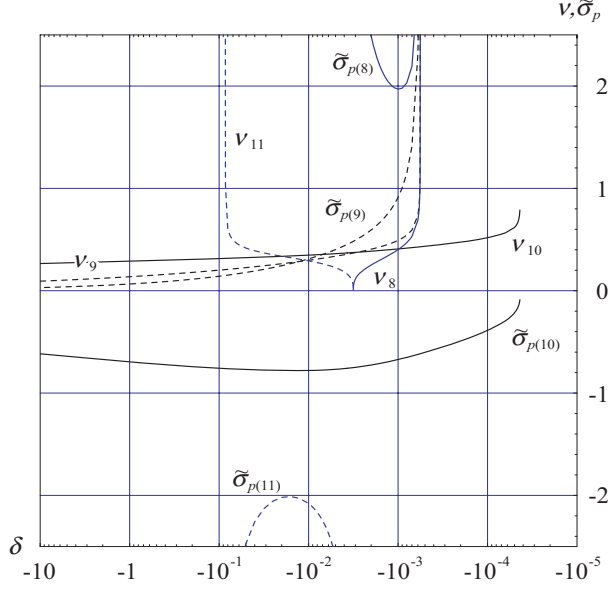


Figure 5: Four Generalized de Sitter solutions with $\sigma_p \neq 0$, $\sigma_q = 0$ with respect to $\delta > 0$. There is no solution for $\delta \geq 0$

We have $A_p = \mu^2$, $A_q = 1 + \tilde{\sigma}_q$, $X = \mu(\mu - 1)$ and $Y = 0$.

For $t \rightarrow \infty$, the EH term dominates and the solution is

$$(\mu, \nu, \tilde{\sigma}_q) = \text{ME12}(0, 1, -1) . \quad (4.9)$$

Actually this is an exact solution for all δ .

For $t \rightarrow -\infty$, fourth-order terms dominate. Our basic Eqs. (2.17) and (2.19) give the same Eqs. (3.18) and (3.19) in § 3.2.2 with $\delta \neq 0$. We get the following two solutions for $\delta = -0.001$

$$(\delta, \mu, \nu, \tilde{\sigma}_q) = \text{MP18}(-0.001, 32.4979, 1, 482.327) , \quad \text{MP19}(-0.001, -47.5769, 1, -172.534) , \quad (4.10)$$

while the following for $\delta = -0.1$

$$(\delta, \mu, \nu, \tilde{\sigma}_q) = \text{MP14}(-0.1, 14.0811, 1, 92.0567) , \quad \text{MP15}(-0.1, -31.2288, 1, -201.029) , \quad (4.11)$$

For the case of $\sigma_p \neq 0$ and $\sigma_q = 0$, exchanging μ, p and ν, q , we obtain one exact solution ME13 and eleven past asymptotic solutions MP10 – MP17 in Eq. (4.3) and the following solutions MP20 – MP 22 for $\delta = -0.001$, while one exact solution ME13 and nine past asymptotic solutions MP8 – MP13 in Eq. (4.4) and the following solutions MP16 – MP18 for $\delta = -0.1$:

$$(\mu, \nu, \sigma_p, a_0) = \text{ME13}(1, 0, -1, 1) . \quad (4.12)$$

$$(\delta, \mu, \nu, \tilde{\sigma}_p) = \text{MP20}(-0.001, 1, 31.7651, 3563.36) , \quad \text{MP21}(-0.001, 1, 24.9773, 8412.47) , \quad (4.13)$$

$$\text{MP22}(-0.001, 1, -0.84176, -1.25634) , \quad (4.13)$$

$$\text{MP16}(-0.1, 1, 14.0787, 618.521) , \quad \text{MP17}(-0.1, 1, -0.634680, -1.44737) , \quad (4.14)$$

$$\text{MP18}(-0.1, 1, -279.628, -1.00255 \times 10^6) . \quad (4.14)$$

4.3 $\sigma_p \sigma_q \neq 0$

4.3.1 Generalized de Sitter solutions ($\epsilon = 0$)

If $\mu = \nu = 0$, our basic equations reduce to Eqs. (3.23) – (3.25). We find again that there is no solution for $\delta \neq 0$. If either $\mu = 0$ or $\nu = 0$ and the other is nonzero, it is clear that there is no exact solution. For asymptotic solutions, we can search them by setting $A_p = \mu^2$, $A_q = \tilde{\sigma}_q$, $X = \mu^2$ and $Y = 0$ for the latter case. This case is actually the same as § 4.2.1, and thus Eqs. (4.5) and (4.6) give asymptotic solutions with $\nu = 0$. For $\delta = -0.001$, we have the following future and past asymptotic solutions:

$$(\delta, \mu, \nu, \tilde{\sigma}_q) = (-0.001, \pm 0.775\,074, 0, 0.278\,981) : \text{MF2(MP2)} \quad \text{for } t \rightarrow \pm\infty . \quad (4.15)$$

For $\delta = -0.1$, we have

$$(\delta, \mu, \nu, \tilde{\sigma}_q) = (-0.1, \pm 0.307\,198, 0, 0.047\,1560) : \text{MF2(MP2)} \quad \text{for } t \rightarrow \pm\infty . \quad (4.16)$$

The first case is obtained by exchanging p, μ and q, ν . From Eqs. (4.7) and (4.8), the solutions are

$$\begin{aligned} (\delta, \mu, \nu, \tilde{\sigma}_p) &= (-0.001, 0, \pm 0.491\,829, 0.911\,910) : \text{MF3(MP3)} , \\ &(-0.001, 0, \pm 0.401\,567, 1.970\,26) : \text{MF4(MP4)} , \\ &(-0.001, 0, \pm 0.408\,387, -0.672\,260) : \text{MF5(MP5)} \quad \text{for } t \rightarrow \pm\infty , \end{aligned} \quad (4.17)$$

for $\delta = -0.001$ and

$$\begin{aligned} (\delta, \mu, \nu, \tilde{\sigma}_p) &= (-0.1, 0, \pm 0.201\,057, 0.141\,557) : \text{MF3(MP3)} , \\ &(-0.1, 0, \pm 0.314\,065, -0.757\,016) : \text{MF4(MP4)} \quad \text{for } t \rightarrow \pm\infty , \end{aligned} \quad (4.18)$$

for $\delta = -0.1$. Note that Eqs. (4.17) – (4.18) were exact solutions for $\sigma_p = 0$ or $\sigma_q = 0$ in § 4.2.1.

For $\mu, \nu \neq 0$, if our ansatz for solution is imposed, it is easy to see that there is no asymptotic solution if μ and ν are of the opposite signs. If they are of the same sign, either $t \rightarrow +\infty$ or $t \rightarrow -\infty$ gives $A_p \rightarrow \mu^2$, $A_q \rightarrow \nu^2$ and there may be solutions which eventuate in these of § 4.1.1 with the same sign. This implies that the inflationary solutions with positive eigenvalues are obtained for asymptotic infinite future, so that they are not interesting from the cosmological point of view. However, it may be useful to check if there are any solutions of this type. In fact, Eq. (4.1) gives set of asymptotic solutions for $\delta = -0.001$

$$(\delta, \mu, \nu) = (-0.001, \pm 0.402\,934, \pm 0.402\,934) : \text{MF1(MP1)} \quad \text{for } t \rightarrow \pm\infty . \quad (4.19)$$

meanwhile, Eq. (4.2) gives solutions for $\delta = -0.1$ as

$$(\delta, \mu, \nu) = (-0.1, \pm 0.168\,203, \pm 0.168\,203) : \text{MF1(MP1)} \quad \text{for } t \rightarrow \pm\infty , \quad (4.20)$$

Note that Eqs. (4.19) and (4.20) were exact solutions for $\sigma_p, \sigma_q = 0$ in § 4.1.1

4.3.2 Power-law solutions ($\epsilon = 1$)

In this case, we first consider the cases when both μ and ν are not equal to 1.

(1) $\mu > 1$ and $\nu > 1$:

In this case, we have same result (1) in § 3.3.2, and there is no asymptotic solution of our form.

(2) $\mu < 1$ and $\nu < 1$:

As $t \rightarrow \infty$ with EH dominance, we again find no consistent solution. As $t \rightarrow -\infty$ with fourth-order-term dominance, and we have same result in § 4.1.2 with $\mu < 1$ and $\nu < 1$. We obtain the seven asymptotic solutions MP10 – MP15 and MP17 from Eq. (4.3) for $\delta = -0.001$, while three asymptotic solutions MP8 – MP10 from Eq. (4.4) for $\delta = -0.1$.

(3) $\mu > 1$ and $\nu < 1$:

The analysis is almost the same as the case (3) in § 3.3.2, and there is no asymptotic solution of our form for both $t \rightarrow \pm\infty$.

(4) $\mu < 1$ and $\nu > 1$:

The analysis is almost the same as the case (4) in § 3.3.2, and there is no asymptotic solution of our form for both $t \rightarrow \pm\infty$.

Next, we discuss the cases in which one of μ or ν is equal to 1 and the other is not:

(5) $\mu > 1$ and $\nu = 1$:

The analysis is almost the same as the case (5) in § 3.3.2, and there is no asymptotic solution of our form for both $t \rightarrow \pm\infty$.

(6) $\mu < 1$ and $\nu = 1$:

As $t \rightarrow \infty$ with EH dominance, A_p diverges as $\tilde{\sigma}_p e^{2(1-\mu)t}$. There is no asymptotic solution of our form. As $t \rightarrow -\infty$, we again recover the case of $\sigma_p = 0, \sigma_q \neq 0$ with the fourth-order-term dominance and solutions in Eqs. (4.9), (4.10) and (4.11). (Note that Eq. (4.9) was an exact solution for $\sigma_p = 0$.) Choosing those with $\mu < 1$, we get asymptotic power-law solutions

$$(\mu, \nu, \tilde{\sigma}_q) = \text{MP23}(0, 1, -1) , \quad (4.21)$$

for all δ , and MP19 in Eq. (4.10) for $\delta = -0.001$ and MP15 in Eq. (4.11) for $\delta = -0.1$.

(7) $\mu = 1$ and $\nu > 1$:

The analysis is almost the same as the case (5). There is no asymptotic solution.

(8) $\mu = 1$ and $\nu < 1$:

The analysis is almost the same as the case (6), then we find the asymptotic solutions as $t \rightarrow -\infty$, which are the same as the case of $\sigma_p \neq 0, \sigma_q = 0$ given in Eqs. (4.12), (4.13) and (4.14). We have asymptotic power-law solutions

$$(\mu, \nu, \tilde{\sigma}_p) = \text{MP24}(1, 0, -1) , \quad (4.22)$$

for all δ , and MP22 in Eq. (4.13) for $\delta = -0.001$ and MP17 and MP18 in Eq. (4.14) for $\delta = -0.1$.

Finally, we consider the remaining case.

(9) $\mu = 1$ and $\nu = 1$:

Here we have constant $A_p = 1 + \tilde{\sigma}_p$ and $A_q = 1 + \tilde{\sigma}_q$. For $t \rightarrow +\infty$, we have the same solution MF8 in § 3.3.2.

For $t \rightarrow -\infty$ with fourth-order-term dominance, we get same two independent Eqs. (3.32) and (3.33) in § 3.3.2. We find solutions

$$\begin{aligned} (\delta, \tilde{\sigma}_p, \tilde{\sigma}_q) = & \text{MP25}(-0.001, 116.501, 9.455) , & \text{MP26}(-0.001, 33.9247, 10.4469) , \\ & \text{MP27}(-0.001, -0.886\,667, -2.590\,21) , & \text{MP28}(-0.001, -3.020\,49, -1.423\,539) , \end{aligned} \quad (4.23)$$

$$\begin{aligned} & \text{MP25}(-0.1, 19.5872, 6.525\,89) , & \text{MP26}(-0.1, -219.014, 20.5567) , \\ & \text{MP27}(-0.1, -0.448\,085, -2.186\,91) , & \text{MP28}(-0.1, -3.443\,20, -1.582\,47) . \end{aligned} \quad (4.24)$$

We summarize our results obtained in this section in Appendix D. Exact solutions for $\delta = -0.001$ are listed in Table 8, future asymptotic solutions for in Table 9 and past asymptotic solutions for in Table 10. For $\delta = -0.1$, we summarize only exact solutions in Table 11.

5 Stability Analysis of Generalized de Sitter Solutions

Since the exact generalized de Sitter solutions $\text{ME1}_\pm - \text{ME11}_\pm$ obtained in § 3.1.1, § 4.1.1 and § 4.2.1 correspond to fixed points in our dynamical system, we have to analyze their stabilities in order to see which solutions are more generic and also to find interesting cosmological solutions. We have performed a linear perturbation analysis around those fixed points for the solutions. In the following subsections, we classify the result by the signature of σ_p and σ_q .

5.1 $\sigma_p = \sigma_q = 0$

In this case, we have exact solutions $\text{ME1}_\pm - \text{ME5}_\pm$ obtained in § 3.1.1 and § 4.1.1. Setting

$$\frac{du_1^{(i)}}{dt} = \mu_i + A_i e^{\sigma^{(i)} t}, \quad \frac{du_2^{(i)}}{dt} = \nu_i + B_i e^{\sigma^{(i)} t}, \quad (5.1)$$

where $|A_i|, |B_i| \ll 1$, we write down the linear perturbation equations:

$$\begin{aligned} F_A(\mu_i, \nu_i, \sigma^{(i)}) A_i + F_B(\mu_i, \nu_i, \sigma^{(i)}) B_i &= 0, \\ G_A(\mu_i, \nu_i, \sigma^{(i)}) A_i + G_B(\mu_i, \nu_i, \sigma^{(i)}) B_i &= 0, \end{aligned} \quad (5.2)$$

obtained from Eqs. (2.17) and (2.18). Quantities F_A , F_B , G_A and G_B are functions of μ_i , ν_i and $\sigma^{(i)}$ given by

$$\begin{aligned} F_A &= 6\alpha_1 [2\mu + 7\nu] + 120960\alpha_4 \nu^5 [21\mu^2 + 14\mu\nu + \nu^2] \\ &\quad - \frac{56\gamma(\mu - \nu)^3}{30375} \gamma [23652\mu^4 - 568368\mu^3\nu + 694341\mu^2\nu^2 - 301231\mu\nu^3 + 29858\nu^4 \\ &\quad - (16647\mu^2 - 8892\mu\nu + 3313\nu^2)(3\mu + 6\nu + \sigma)\sigma] \\ &\quad - 192\delta(6\mu^2 + 21\mu\nu + 28\nu^2)^2 [24\mu^3 - 21\mu^2\nu - 119\mu\nu^2 - 49\nu^3 \\ &\quad - 3(9\mu^2 + 49\mu\nu + 42\nu^2)\sigma - 3(3\mu + 7\nu)\sigma^2], \end{aligned} \quad (5.3)$$

$$\begin{aligned} F_B &= 42\alpha_1 [\mu + 2\nu] + 846720\alpha_4 \mu \nu^4 (\mu + \nu) [5\mu + \nu] \\ &\quad + \frac{56\gamma(\mu - \nu)^3}{30375} \gamma [\mu(91332\mu^3 - 549471\mu^2\nu + 545397\mu\nu^2 - 209006\nu^3) \\ &\quad - (16647\mu^2 - 8892\mu\nu + 3313\nu^2)(2\mu + 7\nu + \sigma)\sigma] \\ &\quad - 1344\delta(6\mu^2 + 21\mu\nu + 28\nu^2)^2 [\mu(3\mu^2 - 9\mu\nu - 49\nu^2) - (6\mu^2 + 45\mu\nu + 49\nu^2)\sigma - (3\mu + 7\nu)\sigma^2], \end{aligned} \quad (5.4)$$

$$\begin{aligned} G_A &= 4\alpha_1 [3\mu + 7\nu + \sigma] + 80640\alpha_4 \nu^5 (3\mu + \nu) [3\mu + 7\nu + \sigma] \\ &\quad - \frac{56\gamma(\mu - \nu)^2(3\mu + 7\nu + \sigma)}{91125} [23652\mu^4 - 550629\mu^3\nu + 707712\mu^2\nu^2 - 320185\mu\nu^3 + 39838\nu^4 \\ &\quad - (16647\mu^2 - 8892\mu\nu + 3313\nu^2)(3\mu + 7\nu + \sigma)\sigma] \\ &\quad - 64\delta(6\mu^2 + 21\mu\nu + 28\nu^2)^2 (3\mu + 7\nu + \sigma) [24\mu^2 + 21\mu\nu - 56\nu^2 - 9(3\mu + 7\nu + \sigma)\sigma], \end{aligned} \quad (5.5)$$

$$\begin{aligned} G_B &= 14\alpha_1 [2\mu + 8\nu + \sigma] + 40320\alpha_4 \nu^4 (15\mu^2 + 12\mu\nu + \nu^2) [2\mu + 8\nu + \sigma] \\ &\quad + \frac{56\gamma(\mu - \nu)^2}{91125} [232605\mu^5 + 27765\mu^4\nu - 3758859\mu^3\nu^2 + 4294423\mu^2\nu^3 - 1845390\mu\nu^4 + 53336\nu^5 \\ &\quad - (8550\mu^4 + 1094319\mu^3\nu + 80985\mu^2\nu^2 - 156303\mu\nu^3 + 178861\nu^4)\sigma \\ &\quad - (16647\mu^2 - 8892\mu\nu + 3313\nu^2)(5\mu + 15\nu + \sigma)\sigma^2] \\ &\quad - 448\delta(6\mu^2 + 21\mu\nu + 28\nu^2)^2 [15\mu^3 + 45\mu^2\nu - 56\mu\nu^2 - 224\nu^3 - (15\mu^2 + 111\mu\nu + 196\nu^2)\sigma \\ &\quad - 3(5\mu + 15\nu + \sigma)\sigma^2], \end{aligned} \quad (5.6)$$

for $p = 3$, $q = 7$, where we omit the subscript of μ_i , ν_i and $\sigma^{(i)}$. The condition that Eq. (5.2) has nontrivial solutions for A_i and B_i is

$$F_A(\mu_i, \nu_i, \sigma^{(i)}) G_B(\mu_i, \nu_i, \sigma^{(i)}) - F_B(\mu_i, \nu_i, \sigma^{(i)}) G_A(\mu_i, \nu_i, \sigma^{(i)}) = 0, \quad (5.7)$$

which yields five modes ($\sigma = \sigma_a^{(i)}$, $a = 1, 2, \dots, 5$) for each solutions $i = 1, \dots, 5$ with fixed $\delta \neq 0$. This is because the basic equations for \dot{u}_1 and \dot{u}_2 are two third-order differential equations plus one constraint which is second order. Eqs. (5.2) are derived from Eqs. (2.17) and (2.18), but we have checked that the results for $\sigma_a^{(i)}$ remain the same if we use any two combinations of Eqs. (2.17) and (2.19). For instance, we have five modes

$$\begin{aligned} \text{ME2}_\pm : \sigma^{(i)} &= (\mp 5.364\,98, \mp 3.844\,14, \mp 3.773\,01, \mp 0.071\,1343, \pm 1.520\,84) , \\ \text{ME3}_\pm : \sigma^{(i)} &= (\mp 4.029\,34, \mp 3.939\,81, \mp 0.089\,5274) \end{aligned} \quad (5.8)$$

for $\delta = -0.001$ and

$$\begin{aligned} \text{ME3}_\pm : \sigma^{(i)} &= (\mp 1.682\,03, \mp 1.609\,88, \mp 0.072\,1502) , \\ \text{ME4}_\pm : \sigma^{(i)} &= (\mp 3.82\,81, \pm 0.072\,7295, \pm 0.876\,493, \pm 0.949\,222, \pm 4.777\,32) , \end{aligned} \quad (5.9)$$

for $\delta = -0.1$. The class of solutions ME3_\pm has only three modes because these solutions are special case with $\mu = \nu$.

For $\delta = 0$, we have only three modes $\sigma^{(i)}_a$ ($a = 1, 2, 3$) because of the conformal invariance of the Weyl tensor. Specifically, we have the following three modes for ME1_\pm

$$\text{ME1}_\pm : \sigma^{(i)} = (\mp 3.871\,09, \pm 6.242\,68, \pm 10.1138) . \quad (5.10)$$

The numbers of stable and unstable modes for various values of δ are summarized in Table 1. The number of unstable modes is important to discuss generality of inflation. For example, for the solution $\text{ME4}_+(\delta, \mu_4, \nu_4)$, there are one stable and four unstable modes. This implies that this solution may not be generic because there are many unstable modes. The probability to approach such generalized de Sitter solution will be very low. On the other hand, the solution ME2_+ has four stable modes as well as one unstable mode. Hence, except for one direction in the phase space, this solution is stable. There may be a finite probability that a generic spacetime first approaches to this solution and eventually evolves into other solution. The solution ME3_+ has three stable modes, which means that this solution is stable against linear perturbations. We find that preferable solutions, that is, the solutions with many stable modes are obtained when its 10-volume expansion rate ($3\mu_i + 7\nu_i$) is positive.

5.2 $\sigma_p = 0, \sigma_q \neq 0$ (or $\sigma_p \neq 0, \sigma_q = 0$)

We have exact solutions ME6_\pm and ME7_\pm obtained in § 4.2.1 for $\sigma_p = 0$ and $\sigma_q \neq 0$. We have also carried out the linear perturbation similar to that in § 5.1. Writing

$$\begin{aligned} \frac{du_1^{(i)}}{dt} &= \mu_i + A_i e^{\sigma^{(i)} t} , \\ u_2^{(i)} &= \ln b_0 + B_i e^{\sigma^{(i)} t} , \end{aligned} \quad (5.11)$$

we derive the linear perturbation equations (5.2). We find

$$\begin{aligned} F_A &= 2p_1\alpha_1\mu + 8\alpha_4\mu[p_7\mu^6 + 3p_5q_1\mu^4\tilde{\sigma}_q + 3p_3q_3\mu^2\tilde{\sigma}_q^2 + p_1q_5\tilde{\sigma}_q^3] \\ &\quad + \frac{4pq_1\gamma(\mu^2 + \tilde{\sigma}_q)^2}{(D-1)^3(D-2)^4} [2N_1(q, p)((p-7)\mu^2 + (p-1)\tilde{\sigma}_q) + (2N_2(p, q) - (p-3)N_3(q, p))(p\mu + \sigma)\sigma] \\ &\quad + 8p\delta\mu((p+1)_0\mu^2 + q_1\tilde{\sigma}_q)^2[(p-7)(p+1)_0\mu^2 + p_1q_1\tilde{\sigma}_q + 6(p^2\mu + \sigma)\sigma] , \\ F_B &= 2\alpha_1[pq\mu\sigma - q_1\tilde{\sigma}_q] - 8\alpha_4[\tilde{\sigma}_q(p_5q_1\mu^6 + 3p_3q_3\mu^4\tilde{\sigma}_q + 3p_1q_5\mu^2\tilde{\sigma}_q^2 + q_7\tilde{\sigma}_q^3) \\ &\quad - \sigma(p_6q\mu^7 + 3p_4q_2\mu^5\tilde{\sigma}_q + 3p_2q_4\mu^3\tilde{\sigma}_q^2 + pq_6\mu\tilde{\sigma}_q^3)] \\ &\quad + \frac{4pq_1\gamma(\mu^2 + \tilde{\sigma}_q)^2}{(D-1)^3(D-2)^4} [2N_1(q, p)\{\tilde{\sigma}_q((5-p)\mu^2 - (p+1)\tilde{\sigma}_q) + (q\mu^3 + (q-6)\mu\tilde{\sigma}_q)\sigma\} \end{aligned} \quad (5.12)$$

$$\begin{aligned}
& + \{2N_2(p, q) - (p-3)N_3(q, p)\} \mu \sigma (\mu - \sigma) (p\mu + \sigma) \\
& + 8\delta((p+1)_0\mu^2 + q_1\tilde{\sigma}_q)^2 [6q\mu\sigma^2(p\sigma + p_1\mu) + p\mu\sigma\{p(p-5)q\mu^2 + (q-6)q_1\tilde{\sigma}_q\} \\
& - q_1\tilde{\sigma}_q\{p(p-5)\mu^2 + q_1\tilde{\sigma}_q\}] ,
\end{aligned} \tag{5.13}$$

$$\begin{aligned}
G_A &= 2p\alpha_1[(p+1)\mu + \sigma] \\
&+ 8\alpha_4((p+1)\mu + \sigma)[p_6\mu^6 + 3p_4(q-1)_2\mu^4\tilde{\sigma}_q + 3p_2(q-1)_4\mu^2\tilde{\sigma}_q^2 + p(q-1)_6\tilde{\sigma}_q^3] \\
&+ \frac{4p(q-1)\gamma((p+1)\mu + \sigma)(\mu^2 + \tilde{\sigma}_q)^2}{(D-1)^3(D-2)^4} [2N_1(q, p)(q\mu^2 + (q-6)\tilde{\sigma}_q) \\
&- \sigma(\sigma + p\mu)(2N_2(p, q) - (p-3)N_3(q, p))] \\
&+ 8p\delta((p+1)\mu + \sigma)((p+1)_0\mu^2 + q_1\tilde{\sigma}_q)^2 [6\sigma(p\mu + \sigma) + (p+1)_0\mu^2 + (q-1)(q-6)\tilde{\sigma}_q] ,
\end{aligned} \tag{5.14}$$

$$\begin{aligned}
G_B &= 2\alpha_1[(q-1)\sigma(p\mu + \sigma) - (q-1)_2\tilde{\sigma}_q] \\
&+ 8\alpha_4[\sigma(p\mu + \sigma)(p_5(q-1)\mu^6 + 3p_3(q-1)_3\mu^4\tilde{\sigma}_q + 3p_2(q-1)_5\mu^2\tilde{\sigma}_q^2 + (q-1)_7\tilde{\sigma}_q^3) \\
&- \tilde{\sigma}_q\{3(p+1)_0(q-1)_6\mu^2\tilde{\sigma}_q^2 + (q-1)_8\tilde{\sigma}_q^3 + (q-1)_4\mu^4((p+1)_3\mu^2 + 3p_2(p^2 - p - 2)\tilde{\sigma}_q)\}] \\
&- \frac{4p(q-1)\gamma(\mu^2 + \tilde{\sigma}_q)^2}{(D-1)^3(D-2)^4} [2N_1(q, p)\{(p+1)\tilde{\sigma}_q((q-2)\mu^2 + (q-8)\tilde{\sigma}_q) \\
&+ \sigma(p\mu + \sigma)(2(2p+q-2)\mu^2 + (p+2q-13)\tilde{\sigma}_q)\} \\
&- \sigma(p\mu + \sigma)\{2N_2(p, q)\sigma(p\mu + \sigma) - N_3(q, p)((1-p^2)\mu^2 + (p-3)\sigma(p\mu + \sigma))\}] \\
&+ 8\delta((p+1)_0\mu^2 + q_1\tilde{\sigma}_q)^2 [6q\sigma^2(p\mu + \sigma)^2 + \sigma(p\mu + \sigma)\{(q-1)(p+1)_0\mu^2 + q_1(q-13)\tilde{\sigma}_q\} \\
&- (p+1)_0(q-1)_2\mu^2\tilde{\sigma}_q - q_1(q-8)(q-1)\tilde{\sigma}_q^2]
\end{aligned} \tag{5.15}$$

The condition for the existence of nontrivial solutions (5.7) yields six modes ($\sigma = \sigma_a^{(i)}$, $a = 1, 2, \dots, 6$) for each solution with fixed δ because we have the new variable u_1 in addition to the variables in § 5.1. For instance, we have five modes

$$\begin{aligned}
\text{ME6}_\pm : \sigma^{(i)} &= (\mp 3.305\,74, \mp 2.177\,00, \mp 1.162\,61 \mp 1.516\,84i, \\
&\mp 1.162\,61 \pm 1.516\,84i, \mp 0.148\,218, \pm 0.980\,516) ,
\end{aligned} \tag{5.16}$$

for $\delta = -0.001$, while

$$\begin{aligned}
\text{ME6}_\pm : \sigma^{(i)} &= (\mp 1.342\,73, \mp 0.770\,998, \mp 0.460\,796 \mp 8.676\,06i, \\
&\mp 0.460\,796 \pm 8.676\,06i, \mp 0.150\,595, \pm 0.421\,134) ,
\end{aligned} \tag{5.17}$$

for $\delta = -0.1$.

For $\sigma_p \neq 0$ and $\sigma_q = 0$, we have four exact solutions $\text{ME9}_\pm - \text{ME11}_\pm$ obtained in § 4.2.1. Exchanging μ, p and ν, q , we obtain the following six modes for each solution:

$$\begin{aligned}
\text{ME8}_\pm : \sigma^{(i)} &= (\mp 4.159\,33, \mp 3.348\,59, \mp 1.721\,40 \mp 2.744\,09i, \\
&\mp 1.721\,40 \pm 2.744\,09i, \mp 0.094\,2078, \pm 0.716\,524) , \\
\text{ME9}_\pm : \sigma^{(i)} &= (\mp 3.115\,29 \mp 1.707\,36i, \mp 3.115\,29 \pm 1.707\,36i, \mp 2.719\,19, \\
&\mp 0.091\,7844, \pm 0.304\,322 \mp 1.707\,36i, \pm 0.304\,322 \pm 1.707\,36i) , \\
\text{ME10}_\pm : \sigma^{(i)} &= (\mp 3.304\,07, \mp 1.429\,35 \mp 0.921\,414i, \mp 1.429\,35 \pm 0.921\,414i, \\
&\mp 1.429\,35 \mp 2.731\,49i, \mp 1.429\,35 \pm 2.731\,49i, \pm 0.445\,363) \quad \text{for } \delta = -0.001 ,
\end{aligned} \tag{5.18}$$

$$\begin{aligned}
\text{ME9}_\pm : \sigma^{(i)} &= (\mp 1.733\,58, \mp 1.319\,40, \mp 0.703\,698 \mp 15.1267i, \\
&\mp 0.703\,698 \pm 15.1267i, \mp 0.087\,9931, \mp 0.326\,182) , \\
\text{ME10}_\pm : \sigma^{(i)} &= (\mp 2.555\,18, \mp 2.408\,32 \mp 1.076\,03i, \mp 2.408\,32 \pm 1.076\,03i, \\
&\pm 0.209\,864 \mp 1.076\,03i, \pm 0.209\,864 \pm 1.076\,03i, \pm 0.356\,724) , \quad \text{for } \delta = -0.1 .
\end{aligned} \tag{5.19}$$

The numbers of stable and unstable modes for other values of δ are summarized in Table 2. Solution ME6₊ has five stable modes and one unstable mode, while ME7₊ has four stable modes and two unstable modes.

6 Solutions in Type-II Superstring

We can discuss the case of type-II superstring in the same manner as M-theory if we keep the dilaton field constant and ignore the contributions from other fields. The low-energy effective actions for type-IIA and IIB superstrings with tree and one-loop corrections are [16]

$$S_{\text{IIA}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} [R + \alpha'^3 \alpha_{\text{II}} \tilde{E}_8 + \alpha'^3 \gamma_{\text{II}} L_W] , \quad (6.1)$$

$$S_{\text{IIB}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} [R + \alpha'^3 \gamma_{\text{II}} L_W] , \quad (6.2)$$

where α_{II} and γ_{II} are given by

$$\alpha_{\text{II}} = \frac{\pi^2 g_s^2}{3^2 \cdot 2^8} , \quad \gamma_{\text{II}} = \frac{\zeta(3)}{2^3} + \frac{\pi^2 g_s^2}{3 \cdot 2^3} . \quad (6.3)$$

There is additional tree-level term L_W with coefficient $\zeta(3)/2^3$ in type-IIA superstring which vanishes under the de-compactification limit and does not exist in the M-theory action (2.1). The one-loop corrections in type-IIA superstring, i.e. \tilde{E}_8 and the rest of L_W , are lifted to $D = 11$ in the limit $g_s \rightarrow \infty$ and in agreement with M-theory corrections (2.5) and (2.6). We can again rescale α_{II} and γ_{II} as Eq. (2.25), and have

$$\tilde{\alpha}_{\text{II}} = \frac{1}{3 \cdot 2^5} \left(1 + \frac{3\zeta(3)}{\pi^2 g_s^2} \right)^{-1} , \quad \tilde{\gamma}_{\text{II}} = 1 . \quad (6.4)$$

In the type-IIA superstring theory, we have a parameter $0 < \tilde{\alpha}_{\text{II}} < 1/(3 \cdot 2^5)$ corresponding to $0 < g_s < +\infty$, whereas $\tilde{\alpha}_{\text{II}} = 0$ in type IIB superstring. In the limit $g_s \rightarrow \infty$, the value of $\tilde{\alpha}_{\text{II}}$ is equivalent to $\tilde{\alpha}_4$ in M-theory.

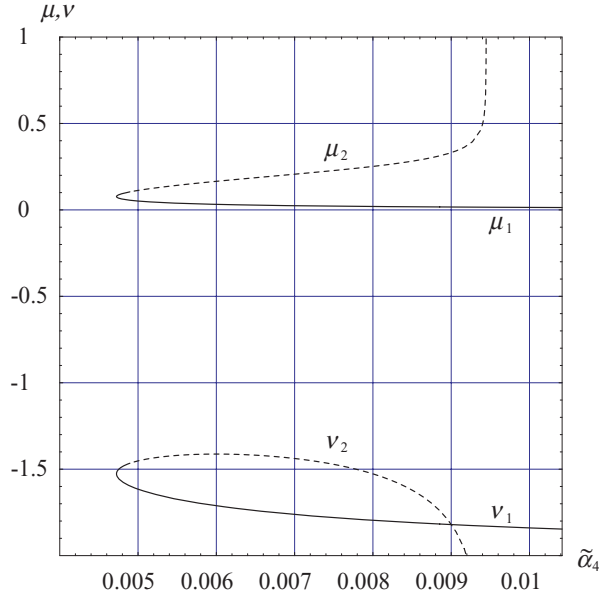


Figure 6: Two generalized de Sitter solutions for type II superstring with $\sigma_p = \sigma_q = 0$ with respect to $0 \leq \tilde{\alpha}_4 \leq 3^{-1}2^{-5}$. $\tilde{\alpha}_4 = 0$ corresponds to the type IIB superstring.

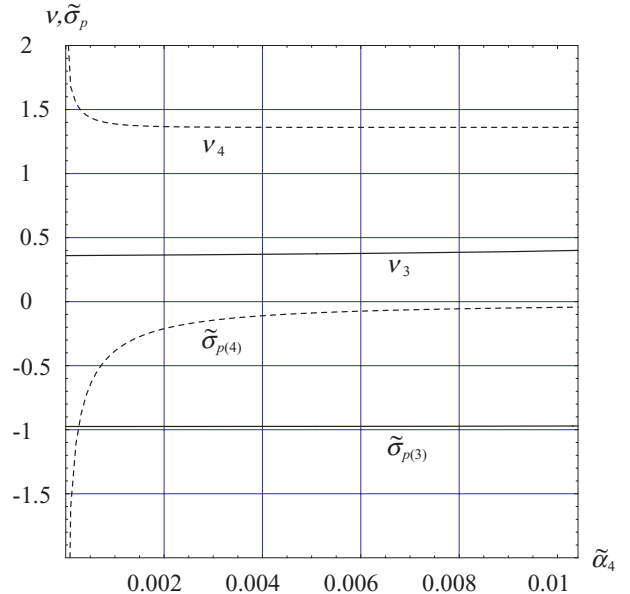


Figure 7: Two Generalized de Sitter solutions for type II superstring with $\sigma_p \neq 0$, $\sigma_q = 0$ with respect to $0 \leq \tilde{\alpha}_4 \leq 3^{-1}2^{-5}$. $\tilde{\alpha}_4 = 0$ corresponds to the type IIB superstring.

We can search the exact and asymptotic solutions in a same way in § 3, however we restrict ourselves to exact generalized de Sitter solutions in the following discussion. We have found four classes of generalized de Sitter solutions for $\sigma_p = \sigma_q = 0$ and $\sigma_p \neq 0, \sigma_q = 0$ in Type IIA superstring while one solution for $\sigma_p \neq 0, \sigma_q = 0$ in Type IIB superstring. There is no solution in other cases.

6.1 $\sigma_p = \sigma_q = 0$

In this case, we have the same Eqs. (3.1) and (3.2) with $\delta = 0$. Exchanging α_4, γ and $\tilde{\alpha}_{\text{II}}, \tilde{\gamma}_{\text{II}}$ and setting $q = 6$, we may solve these equations numerically. In Fig. 6, we give numerical solutions $\text{IIE}i_+(\tilde{\alpha}_{\text{II}}, \mu_i, \nu_i)$ ($i = 1, 2$) with $\mu_i > 0$ with respect to $\tilde{\alpha}_{\text{II}}$, and summarize their properties in Table 3. We have two generalized de Sitter solutions $\text{IIE}1_{\pm}$ and $\text{IIE}2_{\pm}$ for type IIA superstring, whereas no solution for type IIB superstring, i.e. $\tilde{\alpha}_{\text{II}} = 0$.

6.2 $\sigma_p = 0, \sigma_q \neq 0$ (or $\sigma_p \neq 0, \sigma_q = 0$)

It is easy to see that there is no exact solution unless $\nu = 0$ in which case we have constant $A_p = X = \mu^2, A_q = \tilde{\sigma}_q$. Our basic equations reduce to Eqs. (3.15) and (3.16) with $\delta = 0$. Exchanging α_4, γ and $\tilde{\alpha}_{\text{II}}, \tilde{\gamma}_{\text{II}}$ and setting $p = 3, q = 6$, we solve these equations numerically and find that there is no solutions in the range $0 \leq \tilde{\alpha}_{\text{II}} \leq 1/(3 \cdot 2^5)$. Thus we have no solution of this class for both type IIA and type IIB superstring.

For the case of $\sigma_p \neq 0$ and $\sigma_q = 0$, exchanging μ, p and ν, q , we find two solutions $\text{IIE}3_{\pm}$ and $\text{IIE}4_{\pm}$. In Fig. 7, we give numerical solutions $\text{IIE}i_+(\tilde{\alpha}_{\text{II}}, \mu_i, \nu_i)$ ($i = 3, 4$) with $\mu_i > 0$ with respect to $\tilde{\alpha}_{\text{II}}$, and summarize their properties in Table 3.

Table 3: Generalized de Sitter Solutions $\text{IIE}1_{\pm} - \text{IIE}4_{\pm}$ with $\mu_i(\nu_i) \geq 0$ for $0 < g_s < +\infty$. Five eigenmodes for linear perturbations are also shown. (ms, nu) means that there are m stable modes and n unstable modes.

Solution	Property	Range	Stability	$3\mu_i + 7\nu_i$
$\text{IIE}1_+$	$\nu_1 < 0 < \mu_1$	$-0.0047\ 24 < \tilde{\alpha}_4 \leq 3^{-1}2^{-5} \ (0.5506 < g_s < +\infty)$	(1s,2u)	−
$\text{IIE}2_+$	$\nu_2 < 0 < \mu_2$	$-0.0049\ 49 < \tilde{\alpha}_4 < 0.009\ 447 \ (0.5751 < g_s < 1.887)$	(0s,3u)	−
$\text{IIE}3_+$	$\mu_3 = 0, \tilde{\sigma}_{p(3)} < 0 < \nu_3$	$0 \leq \tilde{\alpha}_4 \leq 3^{-1}2^{-5} \ (0 < g_s < +\infty)$	(2s,2u)	+
$\text{IIE}4_+$	$\mu_4 = 0, \tilde{\sigma}_{p(4)} < 0 < \nu_4$	$0 < \tilde{\alpha}_4 \leq 3^{-1}2^{-5} \ (0 < g_s < +\infty)$	(3s,1u)	+

We have two generalized de Sitter solutions $\text{IIE}3_{\pm}$ and $\text{IIE}4_{\pm}$ for type IIA superstring, whereas one solution $\text{IIE}3_{\pm}$ for type IIB superstring.

7 Inflationary Scenario in M-theory

We are ready for discussion about inflation in M-theory. Since our world is 4-dimensional, we should also analyze our solutions in 4-dimensional Einstein frame, in which the Newtonian gravitational constant is constant.

7.1 Description of solutions in the Einstein frame

We have found generalized de Sitter solutions

$$a = a_0 e^{\mu t}, \quad b = b_0 e^{\nu t}, \quad \text{for } \epsilon = 0, \quad (7.1)$$

and power-law solutions

$$a = a_0 \tau^{\mu}, \quad b = b_0 \tau^{\nu}, \quad \text{for } \epsilon = 1. \quad (7.2)$$

These solutions give the power-law expansion and the exponential expansion in the Einstein frame of our 4-dimensional spacetime [23].

- **Generalized de Sitter solutions ($\epsilon = 0$)**

For $\nu \neq 0$, we have the power-law expansion in the Einstein frame.

$$a_E \equiv \exp\left[u_1 + \frac{q}{p-1}u_2\right] \propto t_E^\lambda, \quad \lambda = 1 + \frac{(p-1)\mu}{q\nu}, \quad t_E = t_E^{(0)} \exp\left[\frac{q}{p-1}\nu t\right], \quad (7.3)$$

where for $\nu > 0$, we have $t_E^{(0)} > 0$ and $t \in (-\infty, \infty)$ is transformed into $t_E \in (0, \infty)$, whereas for $\nu < 0$ we have $t_E^{(0)} < 0$ and $t \in (-\infty, \infty)$ is transformed into $t_E \in (-\infty, 0)$. For $\nu = 0$, we have the exponential expansion in the Einstein frame

$$a_E \propto \exp[\mu t_E], \quad t_E = t, \quad (7.4)$$

- **Power-law solutions ($\epsilon = 1$)**

For $\nu \neq -(p-1)/q$, we have the power-law expansion in the Einstein frame

$$a_E \propto t_E^\lambda, \quad \lambda = \frac{(p-1)\mu + q\nu}{(p-1) + q\nu}, \quad t_E = t_E^{(0)} \exp\left[\left(1 + \frac{q}{p-1}\nu\right)t\right], \quad (7.5)$$

where for $\nu > -(p-1)/q$, we have $t_E^{(0)} > 0$ and $t \in (-\infty, \infty)$ is transformed into $t_E \in (0, \infty)$, whereas for $\nu < -(p-1)/q$, we have $t_E^{(0)} < 0$ and $t \in (-\infty, \infty)$ is transformed into $t_E \in (-\infty, 0)$. For $\nu = -(p-1)/q$, we have the exponential expansion in the Einstein frame

$$a_E \propto \exp[(\mu - 1)t_E], \quad t_E = t, \quad (7.6)$$

We list the behavior of a scale factor and show the condition for inflation in the Einstein frame in Table 4. Note that the values of μ and ν in generalized de Sitter solutions (7.1) depend on the choice of the unit. In the M-theory, we use the unit of $|\gamma| = 1$, i.e. $m_{11} = 6^{-1/2}(4\pi)^{-5/9} \sim 0.1818176$. If we set $m_{11} = 1$, the values of μ and ν in the following tables should be multiplied by the factor $6^{1/2}(4\pi)^{5/9} \sim 5.5$. On the other hand, the power exponent μ and ν in the power-law solutions (7.2) or λ in Eqs. (7.3) and (7.5) are dimensionless and they do not depend on the choice of the unit.

Table 4: Behavior of solutions in the Einstein frame; “Condition” means condition for causing inflation in the Einstein frame, while the super-inflationary solution behaves as $|t_E|^{-|\lambda|}$ for $t_E \rightarrow 0_-$.

		Scale Factor	Condition	Range of t_E	Type of inflation
$\epsilon = 0$	$\nu > 0$	$a_E \propto t_E^\lambda$	$\mu/\nu > 0$	$(0, \infty)$	power-law
	$\nu = 0$	$a_E \propto \exp[\mu t_E]$	$\mu > 0$	$(-\infty, \infty)$	exponential
	$\nu < 0$	$a_E \propto t_E^\lambda$	$\lambda < 0$	$(-\infty, 0)$	superinflation
$\epsilon = 1$	$\nu > -(p-1)/q$	$a_E \propto t_E^\lambda$	$\mu > 1$	$(0, \infty)$	power-law
	$\nu = -(p-1)/q$	$a_E \propto \exp[(\mu - 1)t_E]$	$\mu > 1$	$(-\infty, \infty)$	exponential
	$\nu < -(p-1)/q$	$a_E \propto t_E^\lambda$	$\lambda < 0$	$(-\infty, 0)$	superinflation

7.2 Conditions for successful inflation and some preferable solutions

Before we apply our solutions to cosmology, we have to specify what kind of solutions we need. We list necessary conditions for successful inflation in our model below.

- (1) $\mu > \nu$ and $\mu \geq 0$:

Our four-dimensional universe makes sense only if it is much larger than the internal space, so the external space should expand faster than the internal space. Its expansion may not be necessarily inflationary, but at least the external space must be expanding in the whole space. Some solutions give an inflation in the Einstein frame but the external space shrinks in the original higher dimensions. Such solutions are not suitable for a good cosmological model.

(2) 60 e-foldings of inflationary expansion:

We need at least 60 e-foldings of inflationary expansion in the Einstein frame. This may give some constraint on the power exponent for a power-law inflation, that is the power exponent should be significantly larger than unity. Specifically, for generalized de Sitter solution and power-law solution, we find that the number of e-foldings N_E in the Einstein frame is given by

$$N_E = \ln \frac{a_E(t_{\text{fin}})}{a_E(t_{\text{ini}})} = \left(\mu + \frac{q\nu}{p-1} \right) (t_{\text{fin}} - t_{\text{ini}}), \quad (7.7)$$

where t_{ini} is the beginning time of inflation in the original frame whereas t_{fin} is the ending time.

(3) (semi)stability against the linear perturbation:

Solutions which have many unstable modes may not be generic. The most preferable solutions in this model are those with only stable modes, which means that the solution is stable against linear perturbations. However, this kind of solution predicts that inflation never ends because it is stable. For a realistic cosmological model, the solution must have small instability for the inflation to end. On the other hand, we also want such a solution to be rather generic which requires some sort of stability. This would be achieved if the solution contains only one small unstable mode and many other stable modes, and then the generic spacetime may first approach this solution and gradually leave it.

We summarize our solutions in the Einstein frame as well as those in the original frame in Appendix D. Using the tables in Appendix D, we shall pick up the preferable solutions for inflation.

For $\delta = 0$, although we find exponential expansion of the external space in the original frame, this gives non-inflationary power-law expansion in the Einstein frame as described above. There are power-law inflations (MP1, MP11, MP12 and MP13) in the past regime. However, the power exponent of them are 1.3 - 2.3, which may be too small to solve the flatness and horizon problems, because we do not expect the expansion in these solutions continues so long. Thus, these solutions are excluded by the condition (2).

For $\delta = -0.001$, we have six candidates [ME2₊, ME3₊ (MF1), ME6₊ (MF2), MP6 and MP7]. Among these, the condition (1) exclude solutions ME3₊ (MF1) and MP7 for the internal space expands at the same rate as the external space. Solutions ME2₊ and MP6 give super-inflation in the Einstein frame. Especially, the solution ME2₊ has only one unstable mode, and fulfills the condition (3). In this case, we come close to the singularity at $t_E = 0$, but we hope that stringy effects renders the singularity harmless when the curvature becomes large. For the remaining solution ME6₊, we find an exponential expansion of the external space both in the original and the Einstein frames, and the internal space is static, viz. modulus is fixed. From the Table 2, we also find that this solution fulfills the condition (3) and exists for wide range of δ . The radius of the external space is arbitrary whereas that of the internal space is $b_0 = 1.893 = 0.3442 m_{11}^{-1}$.

The above solution ME6₊ has one unstable mode so this is the most interesting cosmological solutions in our criterion. But the instability $\sigma_5^{(i)}$ is the same order of magnitude as other eigenvalues of stable modes as seen from Eq. (5.16) and appears a little too large to give enough expansion. Specifically, the time scale in which this unstable mode becomes important is evaluated as $t_{\text{us}} \sim (\sigma_5^{(6)})^{-1} \sim 1.020$. The number of e-foldings will be given by $N_E = \mu t_{\text{us}} \sim 0.7905$, except for some fine-tuned initial conditions.

If the eigenvalue of the unstable mode is much smaller than those of other four stable modes, however, a preferable solution is naturally obtained for a wide range of initial conditions. We may not need a fine-tuning. Although we do not find any value of δ which gives enough small eigenvalue of an unstable mode, we note that our starting Lagrangian has some ambiguity, that is, the forth-order correction term S_W is fixed up to the Ricci curvature tensors. We have included additional quartic Ricci scalar term (2.7) in order to take this ambiguity into account, but we may still have another kind of correction term including the Ricci curvature tensors. To further improve the solutions, we have the possibility of finding more interesting solutions with these appropriate extra corrections.

Another interesting possibility is the following. We have seen that many of our exact solutions have unstable modes as well as stable modes. But it is not immediately obvious what happens to any inflationary solutions after the solutions decay into other solutions. Our above analysis appears to indicate that we cannot obtain big enough e-foldings before the exact solutions decay. However there is a possibility that we may obtain

enough e-foldings after the decay if we can follow the evolution of our solutions. Indeed we will show in the next section that we obtain some numerical solutions which first approach ME6₊ and give enough e-foldings if we fine-tune the initial data.

7.3 Numerical analysis for generic initial conditions

To study whether we obtain a sufficient e-foldings in the present model, we have performed the numerical calculation around the solution ME6₊ for $u_0 = 0$, $\delta = -0.1$ and $\sigma_q \neq 0$. In this case, Eqs. (2.18) and (2.19) give evolution equations for \ddot{u}_1 and \ddot{u}_2 , and Eq. (2.17) gives a constraint equation for seven variables \ddot{u}_1 , \ddot{u}_2 , \dot{u}_1 , \dot{u}_2 , \ddot{u}_1 , \ddot{u}_2 and u_2 , which we used to check the numerical error. Thus, we can give six independent initial values and the remainder is given through the constraint equation. Under separate five sets of initial conditions, we have performed the numerical calculations for seven dynamical variables.

In Figs. 8 – 11, we depict five numerical solutions MN1 – MN5 whose initial values lie in the vicinity of ME6₊. In Figs. 8 and 9, we show the behavior of solutions in the \dot{u}_1 - \dot{u}_2 plane. Generalized de Sitter solutions are expressed as a point $(\dot{u}_1, \dot{u}_2) = (\mu, \nu)$ in this plane. In these figures, we have the exact solution ME6₊ in Eq. (4.6) and the future asymptotic solution MF1 in Eq. (4.20) (MF1 is exact for $\sigma_q = 0$). Every solution approaches the ME6₊ in the early phase whereas, in the last phase, solutions MN1 – MN4 approach the future asymptotic solution MF1 and the solution MN5 goes to the singularity $\dot{u}_1, \dot{u}_2 \rightarrow -\infty$ for finite lengths of time as shown in Fig. 9.

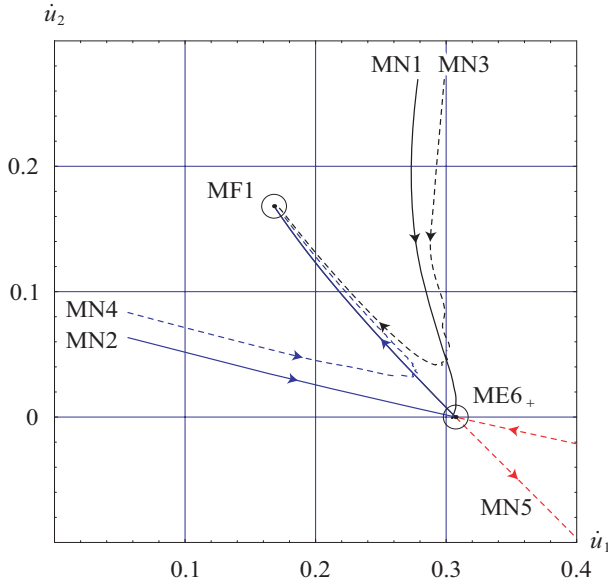


Figure 8: Numerical results around ME6₊ in the \dot{u}_1 - \dot{u}_2 plane for $\delta = -0.1$. Solutions ME6₊ and MF1 are indicated by a point in a circle.

We give the initial values as follows: $b = 3.7$, $\dot{u}_1 = 0.28$, $\dot{u}_2 = 0.27$, $\ddot{u}_1 = -0.060$, $\ddot{u}_2 = -0.46$, $\ddot{\ddot{u}}_1 = 0.47$ for MN1, $b = 4.4$, $\dot{u}_1 = 0.056$, $\dot{u}_2 = 0.063$, $\ddot{u}_1 = 0.24$, $\ddot{u}_2 = 0.068$, $\ddot{\ddot{u}}_1 = -0.15$ for MN2, same as MN1 except for $\dot{u}_1 = 0.30$ for MN3, same as MN2 except for $\dot{u}_2 = 0.083$ for MN4, $b = 5.0$, $\dot{u}_1 = 1.0$, $\dot{u}_2 = -0.13$, $\ddot{u}_1 = -1.6$, $\ddot{u}_2 = 0.20$, $\ddot{\ddot{u}}_1 = 4.7$ for MN5.

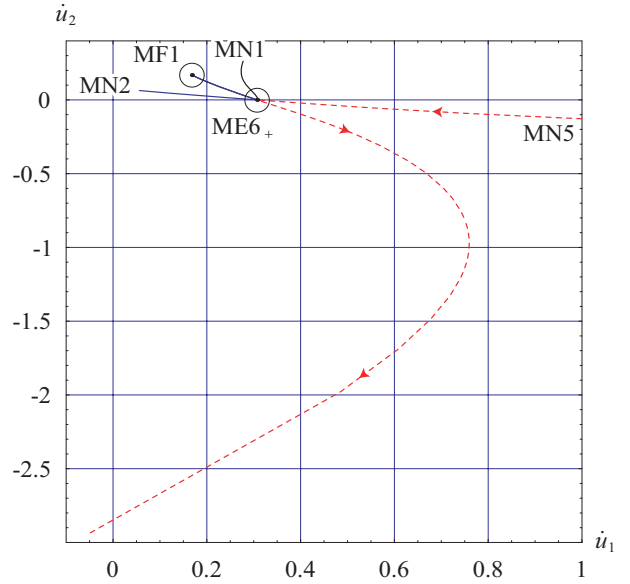


Figure 9: Numerical results around ME6₊ in the \dot{u}_1 - \dot{u}_2 plane. The solution MN5 runs away to $\dot{u}_1, \dot{u}_2 \rightarrow -\infty$ in the last phase.

In Figs 10 and 11, we show the behavior of the scale factors in the original frame (a, b) and in the Einstein frame (a_E, b_E) , respectively. Here we set $a_0 = b_0$ and $t_E(t = 0) = 1$. In the early phase, every solution gives exponential expansion $a_E \propto e^{0.1682t_E}$ arising out of ME6₊ both in the original frame and Einstein frame, while solutions MN1 – MN4 give power-law expansion $a_E \propto t_E^{1.286}$ arising out of MF1 in the Einstein frame in the late phase. The solution MN4 is not shown in the figures since it does not lead to interesting result.

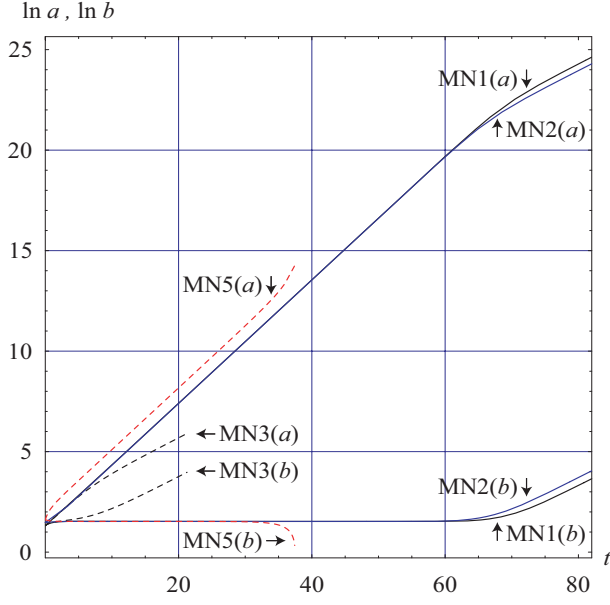


Figure 10: Numerical results in the original frame for $\delta = -0.1$. $MNi(a)$ means the plot of $\ln a$ of the solution MNi . We omit the solution $MN4$.

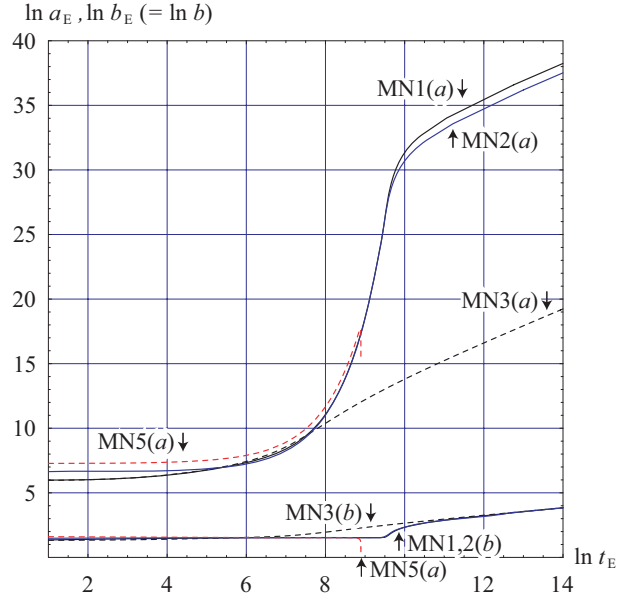


Figure 11: Numerical results in the Einstein frame for $\delta = -0.1$. $MNi(a)$ means the plot of $\ln a_E$ of the solution MNi . We omit the solution $MN4$.

Despite the analysis in § 7.2, we have enough e-foldings around the solution $ME6_+$ with appropriate initial values. Especially, numerical solutions $MN1$ and $MN2$ have interesting features. In the early phase, they give the exponential expansion in the external space, and the power-law expansion in the late phase as shown in Fig. 11. They do not give enough e-foldings around the solution $ME6_+$ ($N_E \sim 30$). However the solution $MF1$, which is a stable solution, does also give inflation. Then after leaving the solution $ME6_+$, a spacetime approaches to the solution $MF1$ in the late phase, which totally gives 60 e-foldings ($N_E = 60$) at $\ln t_E = 35.50$ ($t = 119.7$) for $MN1$ and $\ln t_E = 36.62$ ($t = 119.2$) for $MN2$. The scale of internal space becomes $R_0 = b = 2.182 \times 10^4 = 3967 m_{11}^{-1}$ for $MN1$ and $R_0 = 2.995 \times 10^4 = 5446 m_{11}^{-1}$ for $MN2$ both in the original and Einstein frames. After inflation, if the internal space settles down to static one, the present radius of extra dimensions is given by R_0 . Since this is slightly larger than the fundamental scale length, we may adopt the model of large extra dimensions, which was first proposed as a brane world by Arkani-Hamed et al. [25]. In this model, the 4D Planck mass is given by $m_4^2 = R_0^7 m_{11}^9$. We then find

$$m_{11} = 2.543 \times 10^{-13} m_4 = 619.5 \text{ TeV} , \quad \text{for } MN1 , \quad (7.8)$$

$$m_{11} = 8.392 \times 10^{-14} m_4 = 204.4 \text{ TeV} , \quad \text{for } MN2 . \quad (7.9)$$

This is our fundamental energy scale. The scale of extra dimensions is $R_0 = 3967 m_{11}^{-1} = 6.403 \text{ TeV}^{-1}$ for $MN1$ and $R_0 = 5446 m_{11}^{-1} = 26.64 \text{ TeV}^{-1}$ for $MN2$.

Although numerical solutions $MN1$ and $MN2$ give interesting scenario, their behaviors highly depend on choice of initial values. For example, numerical solutions $MN3$ and $MN4$ have almost the same initial conditions as those of $MN1$ and $MN2$ (see Fig. 8), but their final behaviors are very different (Fig. 11). We can find 60 e-foldings at $\ln t_E = 49.97$ ($t = 83.47$) for $MN3$, and the scale of internal space becomes $R_0 = 1.349 \times 10^6 = 2.463 \times 10^5 m_{11}^{-1}$. Thus, if the extra dimension is so large, the 11D Planck mass turns out to be $m_{11} = 0.3285 \text{ GeV}$, which is excluded by high-energy experiments.

8 Concluding remarks

In this paper, we have examined cosmological solutions of the effective theories of M-theory and superstrings with special attention to the de Sitter-like and power-law expansions. In order to evade the no-go theorem in this setting, we have included higher order quantum corrections. We have also taken the ambiguity arising from the field redefinition into account. We have found that there is an interesting solution $ME6_+$ in which the external space is expanding whereas the size of the internal space is static. This is interesting in another respect that this may be regarded as “moduli stabilization” by the higher order corrections. It is true that this solution does not realize the usual “moduli stabilization” where all moduli are fixed and stable because we have one unstable mode in the fluctuations around this solution. But if the solution were completely stable, the inflation would not end. On the other hand, it is necessary that the size of the internal space does not grow much during inflation. So the feature of this solution is desirable for giving successful inflation. Naively it may be expected that there are many inflationary solutions. It is somewhat surprising to find that we have found few solutions with desirable features.

Quite interestingly, we find some numerical solutions which first approach an exact inflationary solution and then give enough e-foldings if we fine-tune the initial data. The key to understand this result is that we cannot see how the solution evolves after the exact solution decays if we are looking only at the analytic solutions. We were able to follow the evolution by the numerical analysis. Even though it appears that this may need some special initial conditions, we find that this opens another possibility of achieving successful inflation and is a very interesting result.

There is also still ambiguity in other terms involving Ricci tensors and scalar curvatures in the effective theory, which we have not analyzed. So rather than taking our results literally, we should understand these results as an indication that such inflationary solutions are possible in this direction. It is certainly an interesting problem to see if our results may be further improved by including other possible quantum corrections. We also have to look at the effects of fluxes, which play important roles in inflationary models with an effective potential [26]. These may lead to successful inflation insensitive to initial conditions.

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A Explicit form of actions

We use the following notation throughout this paper.

$$\begin{aligned}
(p-m)_n &\equiv (p-m)(p-m-1)(p-m-2)\cdots(p-n), \\
(q-m)_n &\equiv (q-m)(q-m-1)(q-m-2)\cdots(q-n), \\
A_p &\equiv \dot{u}_1^2 + \sigma_p e^{2(u_0-u_1)}, \quad A_q \equiv \dot{u}_2^2 + \sigma_q e^{2(u_0-u_2)}, \\
X &\equiv \ddot{u}_1 - \dot{u}_0 \dot{u}_1 + \dot{u}_1^2, \quad Y \equiv \ddot{u}_2 - \dot{u}_0 \dot{u}_2 + \dot{u}_2^2.
\end{aligned} \tag{A.1}$$

Inputting our ansatz (2.15) into actions (2.2)-(2.9), we can write down the Lagrangians in terms of u_0 , u_1 and u_2 . The explicit forms of actions are listed here, where actions (A.2) – (A.5) are integrated by parts.

(1) Einstein-Hilbert action ($n = 1$)

$$\tilde{E}_2 = e^{-2u_0} [p_1 A_p + q_1 A_q - 2(p_1 \dot{u}_1^2 + p q \dot{u}_1 \dot{u}_2 + q_1 \dot{u}_2^2)] . \tag{A.2}$$

(2) Gauss-Bonnet action ($n = 2$)

$$\begin{aligned}
\tilde{E}_4 &= e^{-4u_0} [p_3 A_p^2 + 2p_1 q_1 A_q A_p + q_3 A_q^2 \\
&\quad - 4A_p(p_3 \dot{u}_1^2 + p_2 q \dot{u}_1 \dot{u}_2 + p_1 q_1 \dot{u}_2^2) - 4A_q(p_1 q_1 \dot{u}_1^2 + p q_2 \dot{u}_1 \dot{u}_2 + q_3 \dot{u}_2^2) \\
&\quad + \frac{4}{3}(2p_3 \dot{u}_1^4 + 2p_2 q \dot{u}_1^3 \dot{u}_2 + 3p_1 q_1 \dot{u}_1^2 \dot{u}_2^2 + 2p q_2 \dot{u}_1 \dot{u}_2^3 + 2q_3 \dot{u}_2^4)] .
\end{aligned} \tag{A.3}$$

(3) Lovelock action ($n = 3, 4$)

$$\begin{aligned}
\tilde{E}_6 &= e^{-6u_0} [p_5 A_q^3 + 3p_3 q_1 A_p^2 A_q + 3p_1 q_3 A_p A_q^2 + q_5 A_q^3 - 6A_p^2(p_5 \dot{u}_1^2 + p_4 q \dot{u}_1 \dot{u}_2 + p_3 q_1 \dot{u}_2^2) \\
&\quad - 6A_q^2(p_1 q_3 \dot{u}_1^2 + p q_4 \dot{u}_1 \dot{u}_2 + q_5 \dot{u}_2^2) - 12A_p A_q(p_3 q_1 \dot{u}_1^2 + p_2 q_2 \dot{u}_1 \dot{u}_2 + p_1 q_3 \dot{u}_2^2) \\
&\quad + 4A_p(2p_5 \dot{u}_1^4 + 2p_4 q \dot{u}_1^3 \dot{u}_2 + 3p_3 q_1 \dot{u}_1^2 \dot{u}_2^2 + 2p_2 q_2 \dot{u}_1 \dot{u}_2^3 + 2p_1 q_3 \dot{u}_2^4) \\
&\quad + 4A_q(2p_3 q_1 \dot{u}_1^4 + 2p_2 q_2 \dot{u}_1^3 \dot{u}_2 + 3p_1 q_3 \dot{u}_1^2 \dot{u}_2^2 + 2p q_4 \dot{u}_1 \dot{u}_2^3 + 2q_5 \dot{u}_2^4) \\
&\quad + \frac{8}{5}(2p_5 \dot{u}_1^6 + 2p_4 q \dot{u}_1^5 \dot{u}_2 + 5p_3 q_1 \dot{u}_1^4 \dot{u}_2^2 + 5p_2 q_2 \dot{u}_1^3 \dot{u}_2^3 + 5p_1 q_3 \dot{u}_1^2 \dot{u}_2^4 + 2p q_4 \dot{u}_1 \dot{u}_2^5 + 2q_5 \dot{u}_2^6)] , \tag{A.4}
\end{aligned}$$

$$\begin{aligned}
\tilde{E}_8 &= e^{-8u_0} [p_7 A_p^4 + 4p_5 q_1 A_p^3 A_q + 6p_3 q_3 A_p^2 A_q^2 + 4p_1 q_5 A_p A_q^3 + q_7 A_q^7 \\
&\quad - 8A_p^3(p_7 \dot{u}_1^2 + p_6 q \dot{u}_1 \dot{u}_2 + p_5 q_1 \dot{u}_2^2) - 8A_q^3(p_1 q_5 \dot{u}_1^2 + p q_6 \dot{u}_1 \dot{u}_2 + q_7 \dot{u}_2^2) \\
&\quad - 24A_p^2 A_q(p_5 q_1 \dot{u}_1^2 + p_4 q_2 \dot{u}_1 \dot{u}_2 + p_3 q_3 \dot{u}_2^2) - 24A_p A_q^2(p_3 q_3 \dot{u}_1^2 + p_2 q_4 \dot{u}_1 \dot{u}_2 + p_1 q_5 \dot{u}_2^2) \\
&\quad + 8A_p^2(2p_7 \dot{u}_1^4 + 2p_6 q \dot{u}_1^3 \dot{u}_2 + 3p_5 q_1 \dot{u}_1^2 \dot{u}_2^2 + 2p_4 q_2 \dot{u}_1 \dot{u}_2^3 + 2p_3 q_3 \dot{u}_2^4) \\
&\quad + 8A_q^2(2p_3 q_3 \dot{u}_1^4 + 2p_2 q_4 \dot{u}_1^3 \dot{u}_2 + 3p_1 q_5 \dot{u}_1^2 \dot{u}_2^2 + 2p q_6 \dot{u}_1 \dot{u}_2^3 + 2q_7 \dot{u}_2^4) \\
&\quad + 16A_p A_q(2p_5 q_1 \dot{u}_1^4 + 2p_4 q_2 \dot{u}_1^3 \dot{u}_2 + 3p_3 q_3 \dot{u}_1^2 \dot{u}_2^2 + 2p_2 q_4 \dot{u}_1 \dot{u}_2^3 + 2p_1 q_5 \dot{u}_2^4) \\
&\quad - \frac{32}{5}A_p(2p_7 \dot{u}_1^6 + 2p_6 q \dot{u}_1^5 \dot{u}_2 + 5p_5 q_1 \dot{u}_1^4 \dot{u}_2^2 + 5p_4 q_2 \dot{u}_1^3 \dot{u}_2^3 + 5p_3 q_3 \dot{u}_1^2 \dot{u}_2^4 + 2p_2 q_4 \dot{u}_1 \dot{u}_2^5 + 2p_1 q_5 \dot{u}_2^6) \\
&\quad - \frac{32}{5}A_q(2p_5 q_1 \dot{u}_1^6 + 2p_4 q_2 \dot{u}_1^5 \dot{u}_2 + 5p_3 q_3 \dot{u}_1^4 \dot{u}_2^2 + 5p_2 q_4 \dot{u}_1^3 \dot{u}_2^3 + 5p_1 q_5 \dot{u}_1^2 \dot{u}_2^4 + 2p q_6 \dot{u}_1 \dot{u}_2^5 + 2q_7 \dot{u}_2^6) \\
&\quad + \frac{16}{35}(8p_7 \dot{u}_1^8 + 8p_6 q \dot{u}_1^7 \dot{u}_2 + 28p_5 q_1 \dot{u}_1^6 \dot{u}_2^2 + 28p_4 q_2 \dot{u}_1^5 \dot{u}_2^3 + 35p_3 q_3 \dot{u}_1^4 \dot{u}_2^4 \\
&\quad + 28p_2 q_4 \dot{u}_1^3 \dot{u}_2^5 + 28p_1 q_5 \dot{u}_1^2 \dot{u}_2^6 + 8p q_6 \dot{u}_1 \dot{u}_2^7 + 8q_7 \dot{u}_2^8)] . \tag{A.5}
\end{aligned}$$

(4) S_W action

Components of the Weyl tensor defined in (2.10) are given by

$$\begin{aligned}
C^t_{itj} &= \frac{e^{-2u_0}}{(D-1)(D-2)} q B_1 g_{ij}, \\
C^t_{atb} &= -\frac{e^{-2u_0}}{(D-1)(D-2)} p B_1 g_{ab}, \\
C^i_{jkl} &= \frac{e^{-2u_0}}{(D-1)(D-2)} q B_2 (g^i_k g_{jl} - g^i_l g_{jk}), \\
C^a_{bcd} &= \frac{e^{-2u_0}}{(D-1)(D-2)} p B_3 (g^a_c g_{bd} - g^a_d g_{bc}), \\
C^i_{ajb} &= \frac{e^{-2u_0}}{(D-1)(D-2)} B_4 g^i_j g_{ab},
\end{aligned} \tag{A.6}$$

where i, j, \dots run over external space, and a, b, \dots over internal space, respectively, and we have defined

$$\begin{aligned}
B_1 &= (D-3)(X-Y) - (p-1)A_p + (q-1)A_q + (p-q)\dot{u}_1\dot{u}_2, \\
B_2 &= -2(X-Y) + (q+1)A_p + (q-1)A_q - 2q\dot{u}_1\dot{u}_2, \\
B_3 &= 2(X-Y) + (p-1)A_p + (p+1)A_q - 2p\dot{u}_1\dot{u}_2, \\
B_4 &= (p-q)(X-Y) - (p-1)qA_p - p(q-1)A_q + (2pq - p - q)\dot{u}_1\dot{u}_2.
\end{aligned} \tag{A.7}$$

Although we have four quantities B_1 , B_2 , B_3 and B_4 , only two of them are independent. Actually, the tracelessness of the Weyl tensor gives the following relations.

$$B_1 + (p-1)B_2 + B_4 = 0, \quad -B_1 + (q-1)B_3 + B_4 = 0. \tag{A.8}$$

Taking independent variables as B_2 and B_3 in order to consider the case $p=1$ or $q=1$ at the same time, we find that the other variables B_1 and B_4 are given as the following with respect to B_2 and B_3 .

$$B_1 = -\frac{1}{2}[(p-1)B_2 - (q-1)B_3], \quad B_4 = -\frac{1}{2}[(p-1)B_2 + (q-1)B_3] \tag{A.9}$$

Substituting Eqs. (A.9) and (A.6) into the action Eq. (2.6), we have the Lagrangian with respect to B_2 and B_3 as follows.

$$L_W = \frac{pq e^{-7u_0+pu_1+qu_2}}{16(D-1)^4(D-2)^4} [n_{1pq}B_2^4 - 4n_{2pq}B_2^3B_3 + 2n_{3pq}B_2^2B_3^2 - 4n_{2qp}B_2B_3^3 + n_{1qp}B_3^4] \tag{A.10}$$

where we define n_{1pq} , n_{2pq} and n_{3pq} as

$$n_{1pq} = (p^2-1)[-(p-1)^4(p-2) + (p-1)^3(p^2+3p-7)q + 2(p-1)^2(p^2-4p+7) + (p^3-13p^2+59p-71)q^3], \tag{A.11}$$

$$n_{2pq} = (p-1)^2(q-1)[-3p^4+3p^3+p^2+p-2+(p-1)(p^3+2p^2+5)q + 2(p(p-1)^2-4)q^2 + (p-3)(p-5)q^3], \tag{A.12}$$

$$\begin{aligned}
n_{3pq} &= (p-1)(q-1)[p\{p^3(3q^2-18q+23) - p^2(9q^2-16q+13) - p(11p+1) - 3\} \\
&\quad + q\{q^3(3p^2-18p+23) - q^2(9p^2-16p+13) - q(11q+1) - 3\} \\
&\quad + 2\{3(pq)^3 + 5(pq)^2 + 8pq - 3\}],
\end{aligned} \tag{A.13}$$

while n_{1qp} and n_{2qp} are given by changing p for q in Eqs. (A.11) and (A.12), respectively.

(5) R^4 action

$$L_{R^4} = e^{-7u_0+pu_1+qu_2} [2pX + 2qY + p_1A_p + q_1A_q + 2pq\dot{u}_1\dot{u}_2]^4. \tag{A.14}$$

B Field equations

Taking variation of the actions (A.2)-(A.14), we find the basic equations (2.17) – (2.19), where each term is summarized here according to which action it originates from. The explicit forms of each term in the field equations are listed here:

(1) EH action ($n = 1$) term

$$F_1 = \alpha_1 e^{-u_0} [p_1 A_p + q_1 A_q + 2pq \dot{u}_1 \dot{u}_2] , \quad (\text{B.1})$$

$$f_1^{(p)} = \alpha_1 e^{-u_0} [(p-1)_2 A_p + q_1 A_q + 2(p-1)q \dot{u}_1 \dot{u}_2] , \quad (\text{B.2})$$

$$f_1^{(q)} = \alpha_1 e^{-u_0} [p_1 A_p + (q-1)_2 A_q + 2p(q-1) \dot{u}_1 \dot{u}_2] , \quad (\text{B.3})$$

$$g_1^{(p)} = 2(p-1)\alpha_1 e^{-u_0} , \quad g_1^{(q)} = 2(q-1)\alpha_1 e^{-u_0} , \quad (\text{B.4})$$

$$h_1^{(p)} = 2q\alpha_1 e^{-u_0} , \quad h_1^{(q)} = 2p\alpha_1 e^{-u_0} . \quad (\text{B.5})$$

(2) Lovelock action ($n = 4$) term

$$\begin{aligned} F_4 = & \alpha_4 e^{-7u_0} [p_7 A_p^4 + 4p_5 q_1 A_p^3 A_q + 6p_3 q_3 A_p^2 A_q^2 + 4p_1 q_5 A_p A_q^3 + q_7 A_q^4 \\ & + 8\dot{u}_1 \dot{u}_2 (p_6 q A_p^3 + 3p_4 q_2 A_p^2 A_q + 3p_2 q_4 A_p A_q^2 + p q_6 A_q^3) \\ & + 24\dot{u}_1^2 \dot{u}_2^2 (p_5 q_1 A_p^2 + 2p_3 q_3 A_p A_q + p_1 q_5 A_q^2) + 32\dot{u}_1^3 \dot{u}_2^3 (p_4 q_2 A_p + p_2 q_4 A_q) + 16p_3 q_3 \dot{u}_1^4 \dot{u}_2^4] , \end{aligned} \quad (\text{B.6})$$

$$\begin{aligned} f_4^{(p)} = & \alpha_4 e^{-7u_0} [(p-1)_8 A_p^4 + 4(p-1)_6 q_1 A_p^3 A_q + 6(p-1)_4 q_3 A_p^2 A_q^2 + 4(p-1)_2 q_5 A_p A_q^3 + q_7 A_q^4 \\ & + 8\dot{u}_1 \dot{u}_2 ((p-1)_7 q A_p^3 + 3(p-1)_5 q_2 A_p^2 A_q + 3(p-1)_3 q_4 A_p A_q^2 + (p-1)q_6 A_q^3) \\ & + 24\dot{u}_1^2 \dot{u}_2^2 ((p-1)_6 q_1 A_p^2 + 2(p-1)_4 q_3 A_p A_q + (p-1)_2 q_5 A_q^2) \\ & + 32\dot{u}_1^3 \dot{u}_2^3 ((p-1)_5 q_2 A_p + (p-1)_3 q_4 A_q) + 16(p-1)_4 q_3 \dot{u}_1^4 \dot{u}_2^4] , \end{aligned} \quad (\text{B.7})$$

$$\begin{aligned} f_4^{(q)} = & \alpha_4 e^{-7u_0} [(q-1)_8 A_q^4 + 4(q-1)_6 p_1 A_q^3 A_p + 6(q-1)_4 p_3 A_q^2 A_p^2 + 4(q-1)_2 p_5 A_q A_p^3 + p_7 A_p^4 \\ & + 8\dot{u}_1 \dot{u}_2 ((q-1)_7 p A_q^3 + 3(q-1)_5 p_2 A_q^2 A_p + 3(q-1)_3 p_4 A_q A_p^2 + (q-1)p_6 A_p^3) \\ & + 24\dot{u}_1^2 \dot{u}_2^2 ((q-1)_6 p_1 A_q^2 + 2(q-1)_4 p_3 A_q A_p + (q-1)_2 p_5 A_p^2) \\ & + 32\dot{u}_1^3 \dot{u}_2^3 ((q-1)_5 p_2 A_q + (q-1)_3 p_4 A_p) + 16(q-1)_4 p_3 \dot{u}_1^4 \dot{u}_2^4] , \end{aligned} \quad (\text{B.8})$$

$$\begin{aligned} g_4^{(p)} = & 8(p-1)\alpha_4 e^{-7u_0} [(p-2)_7 A_p^3 + 3(p-2)_5 q_1 A_p^2 A_q + 3(p-2)_3 q_3 A_p A_q^2 + q_5 A_q^3 \\ & + 6\dot{u}_1 \dot{u}_2 ((p-2)_6 q A_p^2 + 2(p-2)_4 q_2 A_p A_q + (p-2)q_4 A_q^2) \\ & + 12\dot{u}_1^2 \dot{u}_2^2 ((p-2)_5 q_1 A_p + (p-2)_3 q_3 A_q) + 8(p-2)_4 q_2 \dot{u}_1^3 \dot{u}_2^3] , \end{aligned} \quad (\text{B.9})$$

$$\begin{aligned} g_4^{(q)} = & 8(q-1)\alpha_4 e^{-7u_0} [(q-2)_7 A_q^3 + 3(q-2)_5 p_1 A_q^2 A_p + 3(q-2)_3 p_3 A_q A_p^2 + p_5 A_p^3 \\ & + 6\dot{u}_1 \dot{u}_2 ((q-2)_6 p A_q^2 + 2(q-2)_4 p_2 A_p A_q + (q-2)p_4 A_p^2) \\ & + 12\dot{u}_1^2 \dot{u}_2^2 ((q-2)_5 p_1 A_q + (q-2)_3 p_3 A_p) + 8(q-2)_4 p_2 \dot{u}_1^3 \dot{u}_2^3] , \end{aligned} \quad (\text{B.10})$$

$$\begin{aligned} h_4^{(p)} = & 8q\alpha_4 e^{-7u_0} [(p-1)_6 A_p^3 + 3(p-1)_4 (q-1)_2 A_p^2 A_q + 3(p-1)_2 (q-1)_4 A_p A_q^2 + (q-1)_6 A_q^3 \\ & + 6\dot{u}_1 \dot{u}_2 ((p-1)_5 (q-1) A_p^2 + 2(p-1)_3 (q-1)_3 A_p A_q + (p-1)(q-1)_5 A_q^2) \\ & + 12\dot{u}_1^2 \dot{u}_2^2 ((p-1)_4 (q-1)_2 A_p + (p-1)_2 (q-1)_4 A_q) + 8(p-1)_3 (q-1)_3 \dot{u}_1^3 \dot{u}_2^3] , \end{aligned} \quad (\text{B.11})$$

$$\begin{aligned} h_4^{(q)} = & 8p\alpha_4 e^{-7u_0} [(q-1)_6 A_q^3 + 3(q-1)_4 (p-1)_2 A_q^2 A_p + 3(q-1)_2 (p-1)_4 A_q A_p^2 + (p-1)_6 A_p^3 \\ & + 6\dot{u}_1 \dot{u}_2 ((q-1)_5 (p-1) A_q^2 + 2(p-1)_3 (q-1)_3 A_p A_q + (q-1)(p-1)_5 A_p^2) \\ & + 12\dot{u}_1^2 \dot{u}_2^2 ((q-1)_4 (p-1)_2 A_q + (q-1)_2 (p-1)_4 A_p) + 8(p-1)_3 (q-1)_3 \dot{u}_1^3 \dot{u}_2^3] . \end{aligned} \quad (\text{B.12})$$

(3) S_W action term

$$F_W = \gamma e^{-pu_1 - qu_2} \left[-7L_W + 2\sigma_p e^{2(u_0 - u_1)} \left((q+1) \frac{\partial L_W}{\partial B_2} + (p-1) \frac{\partial L_W}{\partial B_3} \right) \right]$$

$$+ 2\sigma_q e^{2(u_0-u_2)} \left[(q-1) \frac{\partial L_W}{\partial B_2} + (p+1) \frac{\partial L_W}{\partial B_3} \right] - 2 \frac{d}{dt} \left\{ (\dot{u}_1 - \dot{u}_2) \left(\frac{\partial L_W}{\partial B_2} - \frac{\partial L_W}{\partial B_3} \right) \right\} \Big] , \quad (\text{B.13})$$

$$\begin{aligned} pF_W^{(p)} = & \gamma e^{-pu_1-qu_2} \left[pL_W - 2\sigma_p e^{2(u_0-u_1)} \left\{ (q+1) \frac{\partial L_W}{\partial B_2} + (p-1) \frac{\partial L_W}{\partial B_3} \right\} \right. \\ & - 2 \frac{d}{dt} \left\{ \left(\dot{u}_0 + (q-1)\dot{u}_1 - q\dot{u}_2 \right) \frac{\partial L_W}{\partial B_2} + \left(-\dot{u}_0 + (p+1)\dot{u}_1 - p\dot{u}_2 \right) \frac{\partial L_W}{\partial B_3} \right\} \\ & \left. - 2 \frac{d^2}{dt^2} \left\{ \frac{\partial L_W}{\partial B_2} - \frac{\partial L_W}{\partial B_3} \right\} \right] , \end{aligned} \quad (\text{B.14})$$

$$\begin{aligned} qF_W^{(q)} = & \gamma e^{-pu_1-qu_2} \left[qL_W - 2\sigma_q e^{2(u_0-u_2)} \left\{ (q-1) \frac{\partial L_W}{\partial B_2} + (p+1) \frac{\partial L_W}{\partial B_3} \right\} \right. \\ & - 2 \frac{d}{dt} \left\{ \left(-\dot{u}_0 - q\dot{u}_1 + (q+1)\dot{u}_2 \right) \frac{\partial L_W}{\partial B_2} + \left(\dot{u}_0 - p\dot{u}_1 + (p-1)\dot{u}_2 \right) \frac{\partial L_W}{\partial B_3} \right\} \\ & \left. + 2 \frac{d^2}{dt^2} \left\{ \frac{\partial L_W}{\partial B_2} - \frac{\partial L_W}{\partial B_3} \right\} \right] , \end{aligned} \quad (\text{B.15})$$

where

$$\frac{\partial L_W}{\partial B_2} = \frac{pq e^{-7u_0+pu_1+qu_2}}{4(D-1)^4(D-2)^4} [n_{1pq} B_2^3 - 3n_{2pq} B_2^2 B_3 + n_{3pq} B_2 B_3^2 - n_{2qp} B_3^3] , \quad (\text{B.16})$$

$$\frac{\partial L_W}{\partial B_3} = \frac{pq e^{-7u_0+pu_1+qu_2}}{4(D-1)^4(D-2)^4} [-n_{2pq} B_2^3 + n_{3pq} B_2^2 B_3 - 3n_{2qp} B_2 B_3^2 + n_{1qp} B_3^3] . \quad (\text{B.17})$$

(4) R^4 action term

$$F_{R^4} = \delta e^{-pu_1-qu_2} \left[-7L_{R^4} + 2\sigma_p e^{2(u_0-u_1)} \frac{\partial L_{R^4}}{\partial A_p} + 2\sigma_q e^{2(u_0-u_2)} \frac{\partial L_{R^4}}{\partial A_q} + \frac{d}{dt} \left(\dot{u}_1 \frac{\partial L_{R^4}}{\partial X} + \dot{u}_2 \frac{\partial L_{R^4}}{\partial Y} \right) \right] , \quad (\text{B.18})$$

$$pF_S^{(p)} = \delta e^{-pu_1-qu_2} \left[pL_{R^4} - 2\sigma_p e^{2(u_0-u_1)} \frac{\partial L_{R^4}}{\partial A_p} + \frac{d}{dt} \left((\dot{u}_0 - 2\dot{u}_1) \frac{\partial L_{R^4}}{\partial X} - 2\dot{u}_1 \frac{\partial L_{R^4}}{\partial A_p} - \frac{\partial L_{R^4}}{\partial \dot{u}_1} \right) + \frac{d^2}{dt^2} \left(\frac{\partial L_{R^4}}{\partial X} \right) \right] , \quad (\text{B.19})$$

$$qF_S^{(q)} = \delta e^{-pu_1-qu_2} \left[qL_{R^4} - 2\sigma_q e^{2(u_0-u_2)} \frac{\partial L_{R^4}}{\partial A_q} + \frac{d}{dt} \left((\dot{u}_0 - 2\dot{u}_2) \frac{\partial L_{R^4}}{\partial Y} - 2\dot{u}_2 \frac{\partial L_{R^4}}{\partial A_q} - \frac{\partial L_{R^4}}{\partial \dot{u}_2} \right) + \frac{d^2}{dt^2} \left(\frac{\partial L_{R^4}}{\partial Y} \right) \right] , \quad (\text{B.20})$$

where

$$\begin{aligned} \frac{\partial L_{R^4}}{\partial X} &= e^{-7u_0+pu_1+qu_2} 8p\tilde{R}^3 , & \frac{\partial L_{R^4}}{\partial Y} &= e^{-7u_0+pu_1+qu_2} 8q\tilde{R}^3 , \\ \frac{\partial L_{R^4}}{\partial A_p} &= e^{-7u_0+pu_1+qu_2} 4p_1\tilde{R}^3 , & \frac{\partial L_{R^4}}{\partial A_q} &= e^{-7u_0+pu_1+qu_2} 4q_1\tilde{R}^3 , \end{aligned} \quad (\text{B.21})$$

$$\frac{\partial L_{R^4}}{\partial \dot{u}_1} = e^{-7u_0+pu_1+qu_2} 8pq\dot{u}_2\tilde{R}^3 , \quad \frac{\partial L_{R^4}}{\partial \dot{u}_2} = e^{-7u_0+pu_1+qu_2} 8pq\dot{u}_1\tilde{R}^3 .$$

$$\tilde{R} = 2pX + 2qY + p_1A_p + q_1A_q + 2pq\dot{u}_1\dot{u}_2 \quad (\text{B.22})$$

C Inputting our ansatz into solutions

In order to find solutions, we assume

$$u_0 = \epsilon t , \quad u_1 = \mu t + \ln a_0 , \quad u_2 = \nu t + \ln b_0 . \quad (\text{C.1})$$

Inserting this form into the above equations (Eqs. (B.1) – (B.15)) and setting

$$A_p = \mu^2 + \tilde{\sigma}_p e^{2(\epsilon-\mu)t}, \quad A_q = \nu^2 + \tilde{\sigma}_q e^{2(\epsilon-\nu)t}, \quad \tilde{\sigma}_p = \frac{\sigma_p}{a_0^2}, \quad \tilde{\sigma}_q = \frac{\sigma_q}{b_0^2}, \quad (\text{C.2})$$

$$X = \mu(\mu - \epsilon), \quad Y = \nu(\nu - \epsilon), \quad (\text{C.3})$$

we obtain the following explicit equations:

(1) EH action ($n = 1$) term

$$\begin{aligned} F_1 &= \alpha_1 e^{-\epsilon t} [p_1 A_p + q_1 A_q + 2pq\mu\nu], \\ f_1^{(p)} &= \alpha_1 e^{-\epsilon t} [(p-1)_2 A_p + q_1 A_q + 2(p-1)q\mu\nu], \\ f_1^{(q)} &= \alpha_1 e^{-\epsilon t} [p_1 A_p + (q-1)_2 A_q + 2p(q-1)\mu\nu], \\ g_1^{(p)} &= 2(p-1)\alpha_1 e^{-\epsilon t}, \quad g_1^{(q)} = 2(q-1)\alpha_1 e^{-\epsilon t}, \\ h_1^{(p)} &= 2q\alpha_1 e^{-\epsilon t}, \quad h_1^{(q)} = 2p\alpha_1 e^{-\epsilon t}, \end{aligned} \quad (\text{C.4})$$

(2) Lovelock action ($n = 4$) term

$$\begin{aligned} F_4 &= \alpha_4 e^{-7\epsilon t} [p_7 A_p^4 + 4p_5 q_1 A_p^3 A_q + 6p_3 q_3 A_p^2 A_q^2 + 4p_1 q_5 A_p A_q^3 + q_7 A_q^4 \\ &\quad + 8\mu\nu(p_6 q A_p^3 + 3p_4 q_2 A_p^2 A_q + 3p_2 q_4 A_p A_q^2 + pq_6 A_q^3) + 24\mu^2 \nu^2 (p_5 q_1 A_p^2 \\ &\quad + 2p_3 q_3 A_p A_q + p_1 q_5 A_q^2) + 32\mu^3 \nu^3 (p_4 q_2 A_p + p_2 q_4 A_q) + 16p_3 q_3 \mu^4 \nu^4], \end{aligned} \quad (\text{C.5})$$

$$\begin{aligned} f_4^{(p)} &= \alpha_4 e^{-7\epsilon t} [(p-1)_8 A_p^4 + 4(p-1)_6 q_1 A_p^3 A_q + 6(p-1)_4 q_3 A_p^2 A_q^2 + 4(p-1)_2 q_5 A_p A_q^3 \\ &\quad + q_7 A_q^4 + 8\mu\nu \{ (p-1)_7 q A_p^3 + 3(p-1)_5 q_2 A_p^2 A_q + 3(p-1)_3 q_4 A_p A_q^2 \\ &\quad + (p-1)_1 q_6 A_q^3 \} + 24\mu^2 \nu^2 \{ (p-1)_6 q_1 A_p^2 + 2(p-1)_4 q_3 A_p A_q + (p-1)_2 q_5 A_q^2 \} \\ &\quad + 32\mu^3 \nu^3 \{ (p-1)_5 q_2 A_p + (p-1)_3 q_4 A_q \} + 16(p-1)_4 q_3 \mu^4 \nu^4], \end{aligned} \quad (\text{C.6})$$

$$\begin{aligned} g_4^{(p)} &= 8(p-1)\alpha_4 e^{-7\epsilon t} [(p-2)_7 A_p^3 + 3(p-2)_5 q_1 A_p^2 A_q + 3(p-2)_3 q_3 A_p A_q^2 + q_5 A_q^3 \\ &\quad + 6\mu\nu \{ (p-2)_6 q A_p^2 + 2(p-2)_4 q_2 A_p A_q + (p-2)_2 q_4 A_q^2 \} \\ &\quad + 12\mu^2 \nu^2 \{ (p-2)_5 q_1 A_p + (p-2)_3 q_3 A_q \} + 8(p-2)_4 q_2 \mu^3 \nu^3], \end{aligned} \quad (\text{C.7})$$

$$\begin{aligned} h_4^{(p)} &= 8q\alpha_4 e^{-7\epsilon t} [(p-1)_6 A_p^3 + 3(p-1)_4 (q-1)_2 A_p^2 A_q + 3(p-1)_2 (q-1)_4 A_p A_q^2 \\ &\quad + (q-1)_6 A_q^3 + 6\mu\nu \{ (p-1)_5 (q-1) A_p^2 + 2(p-1)_3 (q-1)_3 A_p A_q \\ &\quad + (p-1)(q-1)_5 A_q^2 \} + 12\mu^2 \nu^2 \{ (p-1)_4 (q-1)_2 A_p + (p-1)_2 (q-1)_4 A_q \} \\ &\quad + 8(p-1)_3 (q-1)_3 \mu^3 \nu^3], \end{aligned} \quad (\text{C.8})$$

$$\begin{aligned} f_4^{(q)} &= \alpha_4 e^{-7\epsilon t} [(q-1)_8 A_q^4 + 4(q-1)_6 p_1 A_q^3 A_p + 6(q-1)_4 p_3 A_q^2 A_p^2 + 4(q-1)_2 p_5 A_q A_p^3 \\ &\quad + p_7 A_p^4 + 8\mu\nu \{ (q-1)_7 p A_q^3 + 3(q-1)_5 p_2 A_q^2 A_p + 3(q-1)_3 p_4 A_q A_p^2 \\ &\quad + (q-1)_1 p_6 A_p^3 \} + 24\mu^2 \nu^2 \{ (q-1)_6 p_1 A_q^2 + 2(q-1)_4 p_3 A_q A_p + (q-1)_2 p_5 A_p^2 \} \\ &\quad + 32\mu^3 \nu^3 \{ (q-1)_5 p_2 A_q + (q-1)_3 p_4 A_p \} + 16(q-1)_4 p_3 \mu^4 \nu^4], \end{aligned} \quad (\text{C.9})$$

$$\begin{aligned} g_4^{(q)} &= 8(q-1)\alpha_4 e^{-7\epsilon t} [(q-2)_7 A_q^3 + 3(q-2)_5 p_1 A_q^2 A_p + 3(q-2)_3 p_3 A_q A_p^2 + p_5 A_p^3 \\ &\quad + 6\mu\nu \{ (q-2)_6 p A_q^2 + 2(q-2)_4 p_2 A_q A_p + (q-2)_2 p_4 A_p^2 \} \\ &\quad + 12\mu^2 \nu^2 \{ (q-2)_5 p_1 A_q + (q-2)_3 p_3 A_p \} + 8(q-2)_4 p_2 \mu^3 \nu^3], \end{aligned} \quad (\text{C.10})$$

$$\begin{aligned} h_4^{(q)} &= 8p\alpha_4 e^{-7\epsilon t} [(q-1)_6 A_q^3 + 3(q-1)_4 (p-1)_2 A_q^2 A_p + 3(q-1)_2 (p-1)_4 A_q A_p^2 \\ &\quad + (p-1)_6 A_p^3 + 6\mu\nu \{ (q-1)_5 (p-1) A_q^2 + 2(p-1)_3 (q-1)_3 A_q A_p \\ &\quad + (p-1)(q-1)_5 A_p^2 \} + 12\mu^2 \nu^2 \{ (q-1)_4 (p-1)_2 A_q + (p-1)_2 (q-1)_4 A_p \} \\ &\quad + 8(p-1)_3 (q-1)_3 \mu^3 \nu^3], \end{aligned}$$

$$\begin{aligned}
& + (q-1)(p-1)_5 A_p^2 \} + 12\mu^2\nu^2 \{ (q-1)_4(p-1)_2 A_q + (q-1)_2(p-1)_4 A_p \} \\
& + 8(p-1)_3(q-1)_3 \mu^3 \nu^3 \} ,
\end{aligned} \tag{C.11}$$

(3) S_W action term

$$\begin{aligned}
F_W = & \frac{\gamma p q e^{-7\epsilon t}}{16(D-1)^4(D-2)^4} [-7\tilde{L}_W - 2(\mu - \nu)(-7\epsilon + p\mu + q\nu)N_- \\
& + 2\tilde{\sigma}_p e^{2(\epsilon-\mu)t} \{ N_+ + N_- + (\epsilon - \mu)((q+1)N_{-,2} + (p-1)N_{-,3}) \} \\
& + 2\tilde{\sigma}_q e^{2(\epsilon-\nu)t} \{ N_+ - N_- + (\epsilon - \nu)((q-1)N_{-,2} + (p+1)N_{-,3}) \}] .
\end{aligned} \tag{C.12}$$

$$\begin{aligned}
F_W^{(p)} = & \frac{\gamma q e^{-7\epsilon t}}{16(D-1)^4(D-2)^4} [p\tilde{L}_W - 2(-7\epsilon + p\mu + q\nu)((-6\epsilon + (p-1)\mu + q\nu)N_- + (\mu - \nu)N_+) \\
& - 2\tilde{\sigma}_p e^{2(\epsilon-\mu)t} \{ N_+ + N_- + 2(\epsilon - \mu)((-11\epsilon + (2p-3)\mu + 2q\nu)((q+1)N_{-,2} + (p-1)N_{-,3}) \\
& + (\mu - \nu)((q+1)N_{+,2} + (p-1)N_{+,3})) \} \\
& - 4(\epsilon - \nu)\tilde{\sigma}_q e^{2(\epsilon-\nu)t} \{ (-11\epsilon + (2p-1)\mu + 2(q-1)\nu)((q-1)N_{-,2} + (p+1)N_{-,3}) \\
& + (\mu - \nu)((q-1)N_{+,2} + (p+1)N_{+,3}) \} \\
& - 8(\epsilon - \mu)^2 \tilde{\sigma}_p^2 e^{4(\epsilon-\mu)t} \{ (q+1)^2 N_{-,22} + 2(p-1)(q+1)N_{-,23} + (p-1)^2 N_{-,33} \} \\
& - 16(\epsilon - \mu)(\epsilon - \nu)\tilde{\sigma}_p \tilde{\sigma}_q e^{2(2\epsilon-\mu-\nu)t} \{ (q^2 - 1)N_{-,22} + 2(pq+1)N_{-,23} + (p+1)^2 N_{-,33} \} \\
& - 8(\epsilon - \nu)^2 \tilde{\sigma}_q^2 e^{4(\epsilon-\nu)t} \{ (q-1)^2 N_{-,22} + 2(p+1)(q-1)N_{-,23} + (p-1)^2 N_{-,33} \}] ,
\end{aligned} \tag{C.13}$$

$$\begin{aligned}
F_W^{(q)} = & \frac{\gamma p e^{-7\epsilon t}}{16(D-1)^4(D-2)^4} [q\tilde{L}_W + 2(-7\epsilon + p\mu + q\nu)((-6\epsilon + p\mu + (q-1)\nu)N_- + (\mu - \nu)N_+) \\
& + 4(\epsilon - \mu)\tilde{\sigma}_p e^{2(\epsilon-\mu)t} \{ (-11\epsilon + 2(p-1)\mu + (2q-1)\nu)((q+1)N_{-,2} + (p-1)N_{-,3}) \\
& + (\mu - \nu)((q+1)N_{+,2} + (p-1)N_{+,3}) \} \\
& - 2\tilde{\sigma}_q e^{2(\epsilon-\nu)t} \{ N_+ - N_- - 2(\epsilon - \nu)((-11\epsilon + 2p\mu + (2q-3)\nu)((q-1)N_{-,2} + (p+1)N_{-,3}) \\
& + (\mu - \nu)((q-1)N_{+,2} + (p+1)N_{+,3})) \} \\
& + 8(\epsilon - \mu)^2 \tilde{\sigma}_p^2 e^{4(\epsilon-\mu)t} \{ (q+1)^2 N_{-,22} + 2(p-1)(q+1)N_{-,23} + (p-1)^2 N_{-,33} \} \\
& + 16(\epsilon - \mu)(\epsilon - \nu)\tilde{\sigma}_p \tilde{\sigma}_q e^{2(2\epsilon-\mu-\nu)t} \{ (q^2 - 1)N_{-,22} + 2(pq+1)N_{-,23} + (p+1)^2 N_{-,33} \} \\
& + 8(\epsilon - \nu)^2 \tilde{\sigma}_q^2 e^{4(\epsilon-\nu)t} \{ (q-1)^2 N_{-,22} + 2(p+1)(q-1)N_{-,23} + (p-1)^2 N_{-,33} \}] .
\end{aligned} \tag{C.14}$$

where

$$\tilde{L}_W = n_{1pq} B_2^4 - 4n_{2pq} B_2^3 B_3 + 2n_{3pq} B_2^2 B_3^2 - 4n_{2qp} B_2 B_3^3 + n_{1qp} B_3^4 , \tag{C.15}$$

$$M_2 = 4[n_{1pq} B_2^3 - 3n_{2pq} B_2^2 B_3 + n_{3pq} B_2 B_3^2 - n_{2qp} B_3^3] , \tag{C.16}$$

$$M_3 = 4[-n_{2pq} B_2^3 + n_{3pq} B_2^2 B_3 - 3n_{2qp} B_2 B_3^2 + n_{1qp} B_3^3] , \tag{C.17}$$

$$\begin{aligned}
N_+ = & qM_2 + pM_3 \\
= & 4[(qn_{1pq} - pn_{2pq})B_2^3 - (3qn_{2pq} - pn_{3pq})B_2^2 B_3 \\
& + (qn_{3pq} - 3pn_{2qp})B_2 B_3^2 - (qn_{2qp} - pn_{1qp})B_3^3] ,
\end{aligned} \tag{C.18}$$

$$N_- = M_2 - M_3 = 4[(n_{1pq} + n_{2pq})B_2^3 - (3n_{2pq} + n_{3pq})B_2^2 B_3 + (n_{3pq} + 3n_{2qp})B_2 B_3^2 - (n_{2qp} + n_{1qp})B_3^3] , \tag{C.19}$$

$$N_{+,2} = 4[3(qn_{1pq} - pn_{2pq})B_2^2 - 2(3qn_{2pq} - pn_{3pq})B_2 B_3 + (qn_{3pq} - 3pn_{2qp})B_3^2] , \tag{C.20}$$

$$N_{+,3} = 4[-(3qn_{2pq} - pn_{3pq})B_2^2 + 2(qn_{3pq} - 3pn_{2qp})B_2 B_3 - 3(qn_{2qp} - pn_{1pq})B_3^2] , \tag{C.21}$$

$$N_{-,2} = 4[3(n_{1pq} + n_{2pq})B_2^2 - 2(3n_{2pq} + n_{3pq})B_2 B_3 + (n_{3pq} + 3n_{2qp})B_3^2] , \tag{C.22}$$

$$N_{-,3} = 4[-(3n_{2pq} + n_{3pq})B_2^2 + 2(n_{3pq} + 3n_{2qp})B_2 B_3 - 3(n_{2qp} + n_{1qp})B_3^2] , \tag{C.23}$$

$$N_{-,22} = 8[3(n_{1pq} + n_{2pq})B_2 - (3n_{2pq} + n_{3pq})B_3] , \quad (C.24)$$

$$N_{-,23} = 8[-(3n_{2pq} + n_{3pq})B_2 + (n_{3pq} + 3n_{2qp})B_3] , \quad (C.25)$$

$$N_{-,33} = 8[(n_{3pq} + 3n_{2qp})B_2 - 3(n_{2qp} + n_{1qp})B_3^2] . \quad (C.26)$$

(4) R^4 action term

$$F_{R^4} = \delta e^{-7\epsilon t} \tilde{R}^2 [-7\tilde{R}^2 + 8(-7\epsilon + p\mu + q\nu)(p\mu + q\nu)\tilde{R} + 16\tilde{\sigma}_p e^{2(\epsilon-\mu)t} \{p\tilde{R} + 6p_1(\epsilon - \mu)(p\mu + q\nu)\} \\ + 16\tilde{\sigma}_q e^{2(\epsilon-\nu)t} \{q\tilde{R} + 6q_1(\epsilon - \nu)(p\mu + q\nu)\}] , \quad (C.27)$$

$$F_{R^4}^{(p)} = \delta e^{-7\epsilon t} \tilde{R} [\tilde{R}^3 - 8(-7\epsilon + p\mu + q\nu)(6\epsilon + \mu)\tilde{R}^2 \\ + 8(p-1)\tilde{\sigma}_p e^{2(\epsilon-\mu)t} \{-\tilde{R}^2 + 6p(\epsilon - \mu)(-13\epsilon + (p-1)\mu + q\nu)\tilde{R} + 24p(\epsilon - \mu)^2\} \\ + 46q_1(\epsilon - \nu)\tilde{\sigma}_q e^{2(\epsilon-\nu)t} \{(-13\epsilon + (p-1)\mu + q\nu)\tilde{R} + 4(\epsilon - \nu)\} + 192p_1^2(\epsilon - \mu)^2\tilde{\sigma}_p^2 e^{4(\epsilon-\mu)t}\tilde{R} \\ + 384p_1q_1(\epsilon - \mu)(\epsilon - \nu)\tilde{\sigma}_p\tilde{\sigma}_q e^{2(2\epsilon-\mu-\nu)t}\tilde{R} + 192q_1^2(\epsilon - \nu)^2\tilde{\sigma}_q^2 e^{4(\epsilon-\nu)t}\tilde{R}] , \quad (C.28)$$

$$F_{R^4}^{(q)} = \delta e^{-7\epsilon t} \tilde{R} [\tilde{R}^3 - 8(-7\epsilon + p\mu + q\nu)(6\epsilon + \nu)\tilde{R}^2 \\ + 46p_1(\epsilon - \mu)\tilde{\sigma}_p e^{2(\epsilon-\mu)t} \{(-13\epsilon + p\mu + (q-1)\nu)\tilde{R} + 4(\epsilon - \mu)\} \\ + 8(q-1)\tilde{\sigma}_q e^{2(\epsilon-\nu)t} \{-\tilde{R}^2 + 6q(\epsilon - \nu)(-13\epsilon + p\mu + (q-1)\nu)\tilde{R} + 24q(\epsilon - \nu)^2\} \\ + 192p_1^2(\epsilon - \mu)^2\tilde{\sigma}_p^2 e^{4(\epsilon-\mu)t}\tilde{R} + 384p_1q_1(\epsilon - \mu)(\epsilon - \nu)\tilde{\sigma}_p\tilde{\sigma}_q e^{2(2\epsilon-\mu-\nu)t}\tilde{R} \\ + 192q_1^2(\epsilon - \nu)^2\tilde{\sigma}_q^2 e^{4(\epsilon-\nu)t}\tilde{R}] , \quad (C.29)$$

where

$$\tilde{R} = 2pX + 2qY + p_1A_p + q_1A_q + 2pq\mu\nu . \quad (C.30)$$

D Summary Tables of solutions in M-theory

Here we summarize our solutions for $\delta = 0, -0.001$ and -0.1 in tables. In the last columns of the tables, we include the type of two spaces (ds_p^2, ds_q^2) . K means the kinetic dominant space, in which the curvature term (σ_p , or σ_q) is either zero or can be asymptotically ignored. M denotes the Milne-type space, which is described by $ds^2 = -dt^2 + t^2 ds_p^2 + \dots$ with $\sigma_p = -1$, or $ds^2 = -dt^2 + \dots + t^2 ds_q^2$ with $\sigma_q = -1$. Similarly, we define a constant curvature space C by $\sigma_p = 1$ or $\sigma_q = 1$, and S_0 and S_{\pm} are static spaces with zero curvature and positive (or negative) curvature, respectively.

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Table 5: Exact solutions for $\delta = 0$. $\epsilon = 0$ and 1 correspond to generalized de Sitter solutions ($a \sim e^{\mu t}$, $b \sim e^{\nu t}$) and power law ones ($a \sim \tau^\mu$, $b \sim \tau^\nu$), respectively. λ is a power exponent of power law solutions in the Einstein frame ($a_E \sim t_E^\lambda$). K, S_\pm , S_0 , and M mean a kinetic dominance, a static space with positive (or negative) curvature, a flat static space, and a Milne-type space, respectively. There is no inflationary solution in the Einstein frame.

Solution	ϵ	σ_p	σ_q	μ	ν	a_0	b_0	λ	ϕ_1	Type
ME1 $_{\pm}$	0	0	0	± 0.1047	∓ 0.9367	\dots	\dots	0.9681	∓ 0.2676	K K
ME12	1	0	-1	0	1	\dots	1	0.7778	0.7778	S_0 M
ME13	1	-1	0	1	0	1	\dots	1	0	M S_0

Table 6: Future asymptotic solutions ($t \rightarrow \infty$) for $\delta = 0$. There is no inflationary solution in the Einstein frame.

Solution	ϵ	σ_p	σ_q	μ	ν	a_0	b_0	λ	ϕ_1	t_E	Type
MF6	1	0	0	0.5583	-0.0964	\dots	\dots	0.3333	-0.1455	$\rightarrow \infty$	Kasner
MF7	1	0	0	-0.3583	0.2964	\dots	\dots	0.3333	0.1455	$\rightarrow \infty$	Kasner
MF8	1	-1	-1	1	1	0.4714	0.8165	1	0.2222	$\rightarrow \infty$	M M

Table 7: Past asymptotic solutions ($t \rightarrow -\infty$) for $\delta = 0$. MP1, MP4, MP5, MP11, MP12 and MP13 are inflationary solutions in the Einstein frame.

Solution	ϵ	σ_p	σ_q	μ	ν	a_0	b_0	λ	ϕ_1	t_E	Type
MP1	1	0	0, ± 1	1.588	0.3193	\dots	\dots	1.278	0.1508	~ 0	K K
MP2	1	0, ± 1	0, ± 1	0.7336	0.082 88	\dots	\dots	0.7935	0.064 25	~ 0	K K
MP3	1	0, ± 1	0, ± 1	0.7225	-0.1667	\dots	\dots	0.3335	-0.4004	~ 0	K K
MP4	1	0, ± 1	0, ± 1	0.6221	-0.4004	\dots	\dots	1.942	0.9975	~ 0	K K
MP5	1	0, ± 1	0, ± 1	0.1003	-1.701	\dots	\dots	1.182	0.3434	~ 0	K K
MP6	1	0, ± 1	0, ± 1	0.022 04	0.9906	\dots	\dots	0.7811	0.2218	~ 0	K K
MP7	1	0, ± 1	0, ± 1	-0.030 14	0.6209	\dots	\dots	0.6754	0.1957	~ 0	K K
MP8	1	0, ± 1	0, ± 1	-0.3353	0.8502	\dots	\dots	0.6641	0.2139	~ 0	K K
MP9	1	0, ± 1	0, ± 1	-0.6685	0.6343	\dots	\dots	0.4818	0.1970	~ 0	K K
MP10	1	0, ± 1	0	-0.9380	2.573	\dots	\dots	0.8063	0.2571	~ 0	K K
MP11	1	0	-1	6.680	1	\dots	\dots	2.262	0.2222	~ 0	K M
MP12	1	0	-1	6.086	1	\dots	\dots	2.1302	0.2222	~ 0	K M
MP13	1	0	-1	2	1	\dots	\dots	1.222	0.2222	~ 0	K M
MP14	1	0, ± 1	-1	0.4818	1	\dots	\dots	0.8848	0.2222	~ 0	K M
MP15	1	0, ± 1	-1	0.072 89	1	\dots	\dots	0.7940	0.2222	~ 0	K M
MP16	1	1	0, ± 1	1	0.2682	\dots	\dots	1	0.1383	~ 0	M K
MP17	1	1	0, ± 1	1	-9.178	\dots	\dots	1	0.2949	~ 0	M K
MP18	1	-1	0, ± 1	1	0.9318	\dots	\dots	1	0.2187	~ 0	M K
MP19	1	-1	0, ± 1	1	-0.1225	\dots	\dots	1	-0.2144	~ 0	M K
MP23	1	± 1	-1	0	1	1	\dots	0.7778	0.2222	~ 0	S_\pm M
MP24	1	-1	± 1	1	0	\dots	1	1	0	~ 0	M S_\pm
MP25	1	1	-1	1	1	0.6495	0.8898	1	0.2222	~ 0	C M
MP26	1	-1	-1	1	1	0.8413	0.091 06	1	0.2222	~ 0	M M
MP27	1	-1	-1	1	1	0.4780	2.284	1	0.2222	~ 0	M M
MP28	1	-1	-1	1	1	0.4166	0.6216	1	0.2222	~ 0	M M

Table 8: Exact solutions for $\delta = -0.001$. $\epsilon = 0$ and 1 correspond to generalized de Sitter solutions ($a \sim e^{\mu t}$, $b \sim e^{\nu t}$) and power law ones ($a \sim \tau^\mu$, $b \sim \tau^\nu$), respectively. λ is a power exponent of power law solutions in the Einstein frame ($a_E \sim t_E^\lambda$). K, S $_{\pm}$, S $_0$ and M mean a kinetic dominance, a static space with positive (or negative) curvature, a flat static space, and a Milne-type space, respectively. ME3 $_{\pm}$ and ME6 $_+$ are inflationary solution in the Einstein frame, but our 3-space is shrinking in ME3 $_-$.

Solution	ϵ	σ_p	σ_q	μ	ν	a_0	b_0	λ	ϕ_1	Type
ME2 $_{\pm}$	0	0	0	± 1.438	$\mp 0.067\,07$	\dots	\dots	-5.126	$\pm 0.019\,16$	K K
ME3 $_{\pm}$	0	0	0	± 0.4029	± 0.4029	\dots	\dots	1.286	± 0.1151	K K
ME6 $_{\pm}$	0	0	1	± 0.7751	0	\dots	1.893	$e^{\mu t_E}$	0	K S $_+$
ME8 $_{\pm}$	0	1	0	0	± 0.4918	1.0472	\dots	1	± 14.05	S $_+$ K
ME9 $_{\pm}$	0	1	0	0	± 0.4016	1.220	\dots	1	± 0.1167	S $_+$ K
ME10 $_{\pm}$	0	-1	0	0	± 0.4084	0.7124	\dots	1	± 0.1147	S $_-$ K
ME12	1	0	-1	0	1	\dots	1	0.7778	0.2222	S $_0$ M
ME13	1	-1	0	1	0	1	\dots	1	0	M S $_0$

Table 9: Future asymptotic solutions ($t \rightarrow \infty$) for $\delta = -0.001$. MF1 and MF2 are inflationary solution in the Einstein frame.

Solution	ϵ	σ_p	σ_q	μ	ν	a_0	b_0	λ	ϕ_1	t_E	Type
MF1	0	± 1	± 1	0.4029	0.4029	\dots	\dots	1.286	0.1151	$\rightarrow \infty$	ME3 $_+$
MF2	0	± 1	1	0.7751	0	\dots	1.893	$e^{\mu t_E}$	0	$\rightarrow \infty$	ME6 $_+$
MF3	0	1	± 1	0	0.4918	1.0472	\dots	1	14.05	$\rightarrow \infty$	ME8 $_+$
MF4	0	1	± 1	0	0.4016	1.220	\dots	1	0.1167	$\rightarrow \infty$	ME9 $_+$
MF5	0	-1	± 1	0	0.4084	0.7124	\dots	1	0.1147	$\rightarrow \infty$	ME10 $_+$
MF6	1	0	0	0.5583	-0.0964	\dots	\dots	0.3333	-0.1455	$\rightarrow \infty$	Kasner
MF7	1	0	0	-0.3583	0.2964	\dots	\dots	0.3333	0.1455	$\rightarrow \infty$	Kasner
MF8	1	-1	-1	1	1	0.4714	0.8165	1	0.2222	$\rightarrow \infty$	M M

Table 10: Past asymptotic solutions ($t \rightarrow -\infty$) for $\delta = -0.001$. MP1, MP2, MP7, MP9, and MP18 are inflationary solution in the Einstein frame, although our 3-space is shrinking in MP1 and MP2.

Solution	ϵ	σ_p	σ_q	μ	ν	a_0	b_0	λ	ϕ_1	t_E	Type
MP1	0	± 1	± 1	-0.4029	-0.4029	1.286	-0.1151	$\rightarrow -\infty$	ME3 ₊
MP2	0	± 1	1	-0.7751	0	...	1.893	$e^{\mu t_E}$	0	$\rightarrow -\infty$	ME6 ₊
MP3	0	1	± 1	0	-0.4918	1.0472	...	1	-14.05	$\rightarrow -\infty$	ME8 ₊
MP4	0	1	± 1	0	-0.4016	1.220	...	1	-0.1167	$\rightarrow -\infty$	ME9 ₊
MP5	0	-1	± 1	0	-0.4084	0.7124	...	1	-0.1147	$\rightarrow -\infty$	ME10 ₊
MP6	1	0	0, ± 1	121.2	-5.488	-5.603	0.3014	~ 0	K K
MP7	1	0	0	27.08	27.08	1.272	0.2827	~ 0	K K
MP8	1	0	0, ± 1	26.66	-37.15	0.8011	0.2879	~ 0	K K
MP9	1	0	0, ± 1	2.610	-0.1187	3.756	-0.2032	~ 0	K K
MP10	1	0, ± 1	0, ± 1	0.7376	-0.086 31	0.6240	-0.1237	~ 0	K K
MP11	1	0, ± 1	0, ± 1	0.7268	-0.1514	0.4187	-0.3221	~ 0	K K
MP12	1	0, ± 1	0, ± 1	0.1909	0.1395	0.4564	0.093 75	~ 0	K K
MP13	1	0, ± 1	0, ± 1	0.1548	0.1548	0.4519	0.1004	~ 0	K K
MP14	1	0, ± 1	0, ± 1	0.1201	0.1699	0.4483	0.1066	~ 0	K K
MP15	1	0, ± 1	0, ± 1	-0.7576	0.6250	0.4486	0.1961	~ 0	K K
MP16	1	0, ± 1	0	-1.162	1.498	0.6537	0.2399	~ 0	K K
MP17	1	0, ± 1	0, ± 1	-2.408 9	0.5986	-0.1009	0.1934	~ 0	K K
MP18	1	0	1	32.50	1	...	0.045 53	8.000	0.2222	~ 0	K M
MP19	1	0, ± 1	-1	-47.58	1	...	0.076 13	-9.795	0.2222	~ 0	K M
MP20	1	1	0	1	31.77	0.016 75	...	1	0.2832	~ 0	M K
MP21	1	1	0	1	24.98	0.010 90	...	1	0.2825	~ 0	M K
MP22	1	-1	0, ± 1	1	-0.8418	0.8922	...	1	0.4325	~ 0	M K
MP23	1	± 1	-1	0	1	...	1	0.7778	0.2222	~ 0	S _± M
MP24	1	-1	± 1	1	0	1	...	1	0	~ 0	M S _±
MP25	1	1	1	1	1	0.2210	0.3839	1	0.2222	~ 0	C C
MP26	1	-1	1	1	1	0.069 37	0.3495	1	0.2222	~ 0	M C
MP27	1	-1	-1	1	1	1.296	0.6584	1	0.2222	~ 0	M M
MP28	1	-1	-1	1	1	0.5507	0.8095	1	0.2222	~ 0	M M

Table 11: Exact solutions for $\delta = -0.1$. $\epsilon = 0$ and 1 correspond to generalized de Sitter solutions ($a \sim e^{\mu t}$, $b \sim e^{\nu t}$) and power law ones ($a \sim \tau^\mu$, $b \sim \tau^\nu$), respectively. λ is a power exponent of power law solutions in the Einstein frame ($a_E \sim t_E^\lambda$). K, S_±, S₀ and M mean a kinetic dominance, a static space with positive (or negative) curvature, a flat static space, and a Milne-type space, respectively. ME3_± and ME6₊ are inflationary solution in the Einstein frame, but our 3-space is shrinking in ME3₋.

Solution	ϵ	σ_p	σ_q	μ	ν	a_0	b_0	λ	ϕ_1	Type
ME3 _±	0	0	0	± 0.1682	± 0.1682	1.286	± 0.2857	K K
ME4 _±	0	0	0	± 0.7429	∓ 0.4540	0.5326	± 0.2857	K K
ME6 _±	0	0	1	± 0.3072	0	...	4.605	$e^{\mu t}$	0	K S ₊
ME9 _±	0	1	0	0	± 0.2011	2.658	...	1	± 0.2857	S ₊ K
ME10 _±	0	-1	0	0	± 0.3141	1.149	...	1	± 0.2857	S ₋ K
ME12	1	0	-1	0	1	...	1	0.7778	0.2222	S ₀ M
ME13	1	-1	0	1	0	1	...	1	0	M S ₀